Penn Wharton Budget Model: Dynamic OLG Model

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Abstract

Penn Wharton Budget Model (PWBM) seeks to inform policy discussions by combining modern advances in economic modeling, big data science, cloud computing and visualization tools to provide a "sandbox" in which different policy ideas can be tested before legislation is drafted. PWBM's model consists of multiple integrated model components. This document describes the current version of PWBM's Dynamic OLG Model, a dynamic, general equilibrium, incomplete markets, overlapping-generations model.

Contents

1 Overview

The economy consists of a large number of overlapping-generations of households, a perfectly competitive representative firm with constant-returns-to-scale technology and a government with a commitment technology. Time is discrete and one model period corresponds to one year.

2 Variables and parameters

In this section, we define state and parameter variables. We define these upfront to help with exposition. In later discussion, we make the attempt to reiterate definitions to help with the reader's comprehension.

For simplicity, we omit the time subscript when it does not compromise the understanding of the model. We denote next period variables by a prime superscript unless additional clarity is called for. **As a rule, the reader should assume all variables and parameters are time-varying.**

In a steady-state (stationary) equilibrium, the model economy is assumed to be on a balanced-growth path with a population growth rate of g_{pop}. In the following model description, aggregate variables are population aggregates, not per capita values.

As a general rule, upper case letters denote model aggregates, while lower case letters denote household-level variables.

2.1 State variables

Households are heterogeneous with respect to age (j), assets (a), labor productivity (z) and average lifetime labor earnings (b). Let $\mathbf{s} = (j, a, z, b)$ denote a household state. Aggregate state is defined by $\mathbf{S} = (x(\mathbf{s}), K, D)$, where $x(\mathbf{s})$ is the period population density function of households in the economy, K is the capital stock (which may include non-household capital), and D is the government's debt at the beginning of the period. We denote by $X(\mathbf{s})$ the cumulative distribution of households.

We fix the portfolio allocation of a household assets a between capital and debt so that $k(\mathbf{s}, \mathbf{S})$ represents the portion of household assets in physical capital and $d(\mathbf{s}, \mathbf{S})$ represents the portion of assets in government debt. To simplify notation, we write these as k and d.

The derived variables $\psi^{cap}_{port}, \psi^{debt}_{port}$ are the allocations of a between physical capital and government debt. They are determined from aggregate state.

In the case of the small open economy, foreigners hold additional assets such that

$$K = K_f + \underbrace{\int k(s)x(s)ds}_{K_d}, \tag{1}$$

and

$$D = D_f + \underbrace{\int d(s)x(s)ds}_{D_d}, \tag{2}$$

where K_f and D_f represent the amount of capital and debt held by foreigners and K_d and D_d, by domestic households.

It is useful to define

$$I = K' - K(1-\delta), \tag{3}$$

as gross capital investment. Note that this amount includes investment to replace depreciated capital.

2.2 Calibrated variables

Calibrated variables are set so that some moment of the model matches an empirical target. The following are calibrated variables:

- β, γ, and σ are utility function parameters and are set to jointly match targets of capital-output ratio and labor elasticity,
- $\mu_\$$ is the conversion between model price of the numeraire and U.S. dollars; this is calibrated to match GDP per household and is used implicitly to convert statutory dollar amounts to model parameters,
- ν_1, ν_2 are cost of business leverage parameters and are set to match the observed debt-capital ratio of business (as in the literature),
- η is a parameter on the capital adjustment cost function and is set to match literature (or set to zero),
- z is a random variable used to denote labor productivity shocks; its distribution is set to match a particular wage distribution based on empirical data – the labor earnings Gini coefficient.

2.3 Choice variables

Households choose savings, labor, and consumption. In the open economy, foreign capital is such that capital return in the U.S. economy is equalized to its world return. Businesses choose business debt to gain a tax benefit (explained later).

The following is a list of choice variables in the model:

- K'_f is the foreigners' next period choice of capital,
- a' is the household's next period choice of savings into their portfolio,
- c is the household's choice of current period consumption,
- n is the household's choice of current period labor supply, and
- B is the firm's choice of debt.

Government expenditures, taxation, and debt are driven by policy and therefore not considered choice variables.

2.4 Parameters

The model requires a large number of exogeneous input parameters. We adopt some conventions to aid the reader. Parameters of similar type are denoted by a lower-case greek letter with subscripts and superscripts to define the particular application. The following naming conventions are used:

- ζ denotes shares or splits of endogenous variables (e.g., share of corporate income taxed at preferred rates),
- ϕ denotes adjustments of taxable amount to an income or liability (e.g., deductible portion of interest paid),
- θ denotes a tax base or taxable amount,
- τ denotes a tax rate or a tax function, and
- ι denotes an index.

Some indices are exogenous (e.g., CPI) while others are endogenous (e.g., wage growth).

3 Production

There is a representative firm that demands labor services and rents physical capital. The firm production function is given by

$$Y = K^{\alpha} L^{1-\alpha}, \tag{4}$$

where K is aggregate physical capital in the economy, L is aggregate labor, and α is the capital share of income. The capital income share α is an exogenous parameter taken from empirical measures of the capital income share of U.S. GDP. We currently keep α fixed across time.

In the economy, businesses which can be either (a) corporations or (b) pass-through entities. To accommodate this distinction, we split economic production according to an exogenous parameterization produced by the PWBM Tax module. The corporate share is described by the (time-varying) parameter ζ_{corp}^{income} so $K_{corp} = \zeta_{corp}^{income} K$ and $L_{corp} = \zeta_{corp}^{income} L$. Similarly, the pass-through share is defined by $\zeta_{pass}^{income} = 1 - \zeta_{corp}^{income}$. Because of constant returns to scale, splitting the "firm" does not change total production.

Households receive all business income, so the split is made in order to explicitly account for appropriate tax treatment. In the case of corporate income, we apply corporate tax, investment expensing, and interest deductions and only then pass the remainder to the household as a return rate π_{corp}. A similar treatment applies to pass-through income, and we call that distribution rate π_{pass}. Taxation is time-varying to reflect time-based policy changes and compositional changes where applicable.

Under the assumption of perfectly competitive markets, the firm takes market prices as given and have labor and capital demand determined from the marginal products of those inputs.

3.1 Cost of capital

In the model, the cost of capital is determined by tax policy and capital adjustment cost. We assume the output good can be converted uni-directionally into new capital at a rate of one to one. Installing the capital for productive use incurs a cost. For new capital the marginal acquisition cost is 1 unit of the numeraire less the tax subsidy from investment expensing $\phi\tau$, where τ is the marginal tax rate on business income and ϕ is the portion of new investment which is deductible in present value terms, plus the marginal cost to install the next unit of capital. We define a convex capital adjustment cost function for the total cost of gross investment I given a current capital stock K

$$C_K(K, I) = \frac{\eta}{2} \left(\frac{I}{K} \right)^2 K \tag{5}$$

The marginal installation cost is

$$\frac{\partial}{\partial I} C_K(K, I) = \eta \left(\frac{I}{K} \right) \tag{6}$$

Since old and new capital are perfect substitutes in production, their marginal acquisition costs must be identical in equilibrium. If we make the assumption that old capital is already installed, then old capital sells at a premium of $\frac{\partial}{\partial I} C_K(K, I)$ above the cost of converting the output good to new capital (at a price of 1). Because the purchaser of new capital receives an implicit $\phi\tau$ subsidy from the government while the purchaser of old capital receives no tax subsidy, old capital is also discounted by $\phi\tau$ to make capital purchasers indifferent between old and new capital.[1]

In the case of corporate investment, the cost of corporate capital is $1-\phi_{corp}\tau_{corp}^{statutory}+\frac{\partial}{\partial I_{corp}}C_K(K_{corp},I_{corp})$ where the tax rate $\tau_{corp}^{statutory}$ is the statutory tax rate on corporate income. However, because non-corporate capital also exists, we require a more complex formulation. There is no single marginal cost of pass-through capital since pass-through capital income enters into the PIT which is a non-linear function. Moreover, it becomes necessary to reconcile different costs of corporate and pass-through capital. To proceed, we are forced to make additional simplifying assumptions or face having to model a capital trade market. The assumptions are:

1. Pass-through income accrues to individuals who are subject to the highest individual marginal tax rate, call it τ_{pit}^{top}, so the implied cost of pass-through capital is $1-\phi_{pass}\tau_{pit}^{top}+\frac{\partial}{\partial I_{pass}}C_K(K_{pass},I_{pass})$.

2. Pass-through capital and corporate capital are not substitutes.

3. The division of capital between pass-through firms and corporations is fixed exogenously by parameters ζ_{pass}^{income} and ζ_{corp}^{income}.

4. Foreigners price both kinds of capital at the same value as U.S. shareholders.

Under these assumptions, the cost of existing capital becomes

$$q=\zeta_{corp}^{income}\left(1-\tau_{corp}^{statutory}\phi_{corp}+\frac{\partial}{\partial I_{corp}}C_K(K_{corp},I_{corp})\right)+\zeta_{pass}^{income}\left(1-\tau_{pit}^{top}\phi_{pass}+\frac{\partial}{\partial I_{pass}}C_K(K_{pass},I_{pass} \tag{7}$$

Note that the assumption on foreigners pricing capital the same as U.S. households only applies for the small open economy model. Foreigners are not subject to the same taxation as U.S. shareholders so their implied cost of capital is different. Households would prefer to sell their capital to foreigners who value the capital more, but we do not allow this trade so as to avoid having to explicitly model a market for capital.

We take a wealth concept of capital measure in period $t=0$. This measure determines the quantity of capital used as input for the production function and determines the units of capital which are added by investment.

To acquire additional capital in subsequent periods $t\geq 1$, households must pay a price $p_{K,t}$ (where $p_{K,0}=1$). This price is defined to come from the cost of capital. Since the model forces households to reacquire capital for next period, denoted as K', this

[1]This implied market condition only holds so long as $I>0$. If the economy is dis-investing in aggregate, then the market will not clear at the price of new capital.

approach seems consistent. The price is the normalized cost of capital, as follows:

$$p_{K,t} = p_{K,0}\frac{q_t}{q_0}, \tag{8}$$

where q_t is the cost of capital in period t.

3.2 Defining business income

There are two types of businesses: corporate [2] and pass-through . Businesses invest in new capital. Tax law allows businesses to deduct the value of investment over time. Call ϕ_x^{exp} the investment expensing ratio for business of type $x \in \{corp, pass\}$. The value ϕ_x^{inv} is calculated to be the net present value of depreciation deductions per dollar of investment for businesses of type x. Note that this modeling choice implies that deducting the value of investment is equivalent to receiving a one-time investment "subsidy" paid out by the government in the current period, whereas in actuality these reductions in government revenue would occur over many years. We assume the existence of markets which would price these tax consequences for the firm, so there is no distortion to the choice of investment.

Business tax parameters are empirically measured from historical data and projected forward by the PWBM microsimulation. The choice between investing in pass-through versus corporate business is not modeled in the OLG model, which takes these decisions as exogenous. The choice of business entity form is modeled by the PWBM microsimulation and passed via parameters to the OLG model.

3.2.1 Corporate income and taxation

Define $Y_{corp} = K_{corp}^{\alpha} L_{corp}^{1-\alpha}$ as gross corporate sector income. The derivation of the interest rate on debt and its composition are explained later; the firm pays a rate given as:

$$r_{corp}^{debt} = \left(\rho_{corp} B_{corp} + \nu_{corp}\left(\frac{B_{corp}}{K_{corp}^{scale}}\right) K_{corp}^{scale}\right)\frac{1}{B_{corp}}, \tag{9}$$

where B_{corp} is the firm's debt, ρ_{corp} is the interest rate the firm faces, and $\nu_{corp}(\cdot)$ is the additional cost of leverage for corporations. The variable $K_{corp}^{scale} = K_{corp} p_K h$ is a scaled measure of the value of corporate assets. The scaling parameter h is discussed later. The corporation's economic operating expenses are labor expenses, wL_{corp}, capital depreciation, $\delta p_K K$, capital adjustment costs, $C_K(\cdot)$, interest payments, $r_{corp}^{debt} B_{corp}$, and other expenses (e.g., property taxes), denoted by $\zeta_{corp}^{other} Y_{corp}$. While labor expenses and interest payments are delivered to households, capital adjustment costs and other expenses are treated as a loss to the economy. Inasmuch as payments of other expenses accrue to the

[2] By "corporate," we mean entities subject to the corporate income tax. Other corporate entities, such as S corporations, are included with pass-throughs.

federal government, they are accounted for in the tax revenue residual (described later in the section on the government's budget).

Define corporate profits as:

$$\Pi_{corp} = Y_{corp} - wL_{corp} - \delta p_K K_{corp} - C_K(K_{corp}, I_{corp}) - r^{debt}_{corp} B_{corp} - \zeta^{other}_{corp} Y_{corp}. \quad (10)$$

Further, define corporate investment for the purposes of taxation as

$$I^{tax}_{corp} = \max\left\{0, p_K(K'_{corp} - (1-\delta)K_{corp})\right\}, \quad (11)$$

so that dis-investment does not imply a tax payment. Rate of return π_{corp} on business income from corporate capital K_{corp} is defined by:

$$\begin{aligned}
\pi_{corp} p_K K_{corp} =& Y_{corp} && \text{(Output)}\\
& - wL_{corp} && \text{(Labor costs)}\\
& - \delta p_K K_{corp} && \text{(Depreciated K)}\\
& - C_K(K_{corp}, I_{corp}) && \text{(Capital adjustment cost)}\\
& - \zeta^{other}_{corp} Y_{corp} && \text{(Other expenses)}\\
& + \tau^{statutory}_{corp} \phi^{exp}_{corp} I^{tax}_{corp} && \text{(Investment expensing)}\\
& + \tau^{statutory}_{corp} \phi^{int}_{corp} r^{debt}_{corp} B_{corp} && \text{(Interest deduction)}\\
& - \tau^{statutory}_{corp} \left(\zeta^{taxbase}_{corp} \Pi_{corp} - \zeta^{ded}_{corp} Y_{corp}\right) && \text{(Tax, net of other deductions)}\\
& + \zeta^{cred}_{corp} Y_{corp} && \text{(Tax credits)}\\
& - \nu_{corp}\left(\frac{B_{corp}}{K^{scale}_{corp}}\right) K^{scale}_{corp}, && \text{(NPV cost of leverage)}
\end{aligned}$$

where

- p_K is the price of capital,
- $\tau^{statutory}_{corp}$ is the statutory corporate income tax rate,
- $w = MP_L$ is the wage rate,
- δ is the economic depreciation rate of capital,
- $C_K(\cdot)$ is the capital adjustment total cost function,
- ζ^{report}_{corp} is the share of the corporate profits which is subject to taxation,
- ζ^{ded}_{corp} is the share of corporate GDP which is the base for tax deductions,
- ζ^{cred}_{corp} is the share of corporate GDP which is the base for tax credits,

- ϕ_{corp}^{exp} defines the current period value of investment depreciation deductions, and
- ϕ_{corp}^{int} is the deductible share of interest.

Note that the firm does not pay out interest, since it borrows from itself as explained later. Shareholders receive the return to both equity and debt. The "bond payment" from the corporation, $r_{corp}^{debt}B_{corp}$, receives different tax treatment at the shareholder level than the distribution of profits, which is the remainder of $\pi_{corp}p_K K_{corp}$.

From the above, the total amount of tax revenue from corporate taxation is given by

$$\begin{aligned} T_{corp} =& \tau_{corp}^{statutory}\left(\zeta_{corp}^{taxbase}\Pi_{corp} - \zeta_{corp}^{ded}Y_{corp}\right) && \text{(Tax, net of other deductions)} \\ &- \tau_{corp}^{statutory}\phi_{corp}^{exp}I_{corp}^{tax} && \text{(Investment expensing)} \\ &- \tau_{corp}^{statutory}\phi_{corp}^{int}r_{corp}^{debt}B_{corp} && \text{(Interest deduction)} \\ &- \zeta_{corp}^{cred}Y_{corp}. && \text{(Tax credits)} \end{aligned}$$

3.2.2 Pass-through business income

Define $Y_{pass} = K_{pass}^{\alpha}L_{pass}^{1-\alpha}$ as gross pass-through sector income, and the interest rate on pass-through debt is:

$$r_{pass}^{debt} = \left(\rho_{pass}B_{pass} + \nu_{pass}\left(\frac{B_{pass}}{K_{pass}^{scale}}\right)K_{pass}^{scale}\right)\frac{1}{B_{pass}}, \tag{12}$$

where B_{pass} is the firm's debt, ρ_{pass} is the interest rate the firm faces, and $\nu_{pass}(\cdot)$ is the additional cost of leverage for pass-throughs. The variable $K_{pass}^{scale} = K_{pass}p_K h$ is analogous to the the variable K_{corp}^{scale}.

Define pass-through investment for the purposes of taxation as

$$I_{pass}^{tax} = \max\left(0, p_K(K'_{pass} - (1-\delta)K_{pass})\right) \tag{13}$$

so that dis-investment does not imply a tax payment. Pass-through profits are defined as:

$$\begin{aligned} \Pi_{pass} =& Y_{pass} && \text{(Output)} \\ &- wL_{pass} && \text{(Labor expenses)} \\ &- \delta p_K K_{pass} && \text{(Depreciated K)} \\ &- C_K(K_{pass}, I_{pass}) && \text{(Capital adjustment cost)} \\ &- \zeta_{pass}^{other}Y_{pass} && \text{(Other expenses)} \\ &+ \phi_{pass}^{exp}I_{pass}^{tax} && \text{(Investment expensing)} \\ &+ \phi_{corp}^{int}r_{pass}^{debt}B_{pass}. && \text{(Interest deduction)} \end{aligned}$$

As with corporate business income, pass-through debt payments are included in Π_{pass}.

Rate of return π_{pass} on business income from pass-through capital K_{pass} is defined by:

$$\begin{aligned}
\pi_{pass} p_K K_{pass} = & Y_{pass} && \text{(Output)} \\
& - wL_{pass} && \text{(Labor expenses)} \\
& - \delta p_K K_{pass} && \text{(Depreciated K)} \\
& - C_K(K_{pass}, I_{pass}) && \text{(Capital adjustment cost)} \\
& - \zeta_{pass}^{other} Y_{pass} && \text{(Other expenses)} \\
& - \nu_{pass} \left(\frac{B_{pass}}{K_{pass}^{scale}} \right) K_{pass}^{scale}, && \text{(NPV cost of leverage)}
\end{aligned}$$

where the variables are pass-through analogues of those described in the section on corporate income. Taxation of pass-through income occurs at the shareholder level.

3.3 Business debt

We allow businesses to issue single period debt. This debt B is used only as a means for the business entity to claim an interest deduction on taxes. We posit a convex cost of leverage function $\nu(\cdot)$ so that the business can solve first order conditions (FOCs) to find an optimal amount of debt which maximizes the tax benefit. Although tax benefit is the only modelled benefit of debt, other benefits exist (e.g., monitoring of the firm by creditors).

We set the interest rate on debt to be ρ. Bondholders must earn the same after-tax return as capital holders, so, under the assumption that effective taxes on bond and capital income are the same, it must be that

$$\rho = \pi. \tag{14}$$

While it is the case that, at the personal level, tax rates on interest income and capital income differ, it is not the case for tax-deferred or tax-exempt accounts. Further empirical work is needed to identify the actual wedge between ρ and π. In the current model, we make the simplifying assumption that the effective tax rates on both returns are equal.

We can have households own all capital and corporate debt and track all payments individually; however, to simplify the accounting, we assume that the firm itself is the bondholder but that the firm is still bound by constraint (14) for each type of business. In this way, the debt payments are netted out as in the definition of the business income equation in section 3.2. Note that π depends on ρ, so we must find a fixed point. Envelope conditions allow the firm to treat ρ as given in solving for optimal debt.

The cost of leverage is assumed to be the increased cost of borrowing. In a deterministic model, there is no risk-premium, so it is not consistent to have a difference in rates of return. If rates are different, agents would rationally pick the higher rate asset. In order to reconcile a "cost of leverage" with our deterministic model, we map a higher dimension model with risk onto our lower dimensional model. In this mapping, the risk-premium is simply some additional cost which is paid by the firm to guarantee a riskless return ρ. This cost is a dead-weight loss to the economy. Conceptualize investors as receiving a higher risky return but valuing it at the lower return ρ. Because the business actually pays out the higher return of ρ plus the risk-premium, the business realizes a tax benefit on the higher return.

3.3.1 Leverage cost function

The leverage cost function attempts to fit observed leverage ratios under various tax policy and macroeconomic environments. Many unmodelled factors may influence the cost of leverage, such as monetary policy, principal-agent problems with asymmetric information, signalling through debt and equity issuance, and so on. We recognize that this reduced form leverage function abstracts from these effects, since the cost of leverage scales only in the leverage ratio. We pick the functional form for the convex leverage function to be:

$$\nu\left(\frac{B}{K^{scale}}\right) = \frac{1}{\nu_1}\left(\frac{B}{K^{scale}}\right)^{\nu_1}, \tag{15}$$

for some constant ν_1. Capital value is scaled by a parameter ν_2, so $K^{scale} = Kp_K\nu_2$. The parameter ν_1 determines the responsiveness of the cost to the leverage ratio, while the scaling factor ν_2 affects at what point the cost begins to increase more sharply. The price of capital p_K affects the leverage ratio since higher value of assets reduces cost to the bondholder (in case of default). In addition, in our model, p_K changes from policy which affects after-tax returns to equity holders, so higher after-tax returns (from policy which also generates a higher p_K) reduce cost to the bondholder.

The total cost of leverage can be written as the rate $\nu(\cdot)$ multiplied by either (a) debt B or (b) the scaled value of capital K^{scale}. While it may seem more natural to think of leverage cost increasing in debt as in option (a), it is also qualitatively equivalent to scale the total cost of debt by the size of the firm.[3] There is some analytic convenience in using the form (b), so we choose that option.

[3]Notice that holding the leverage ratio fixed implies that a larger firm also has a larger debt.

3.3.2 Solving for optimal debt

The firm chooses B contemporaneously to maximize tax benefit[4] net of leverage cost. We can thus find an analytic equation for the optimal B^* from the FOC

$$\frac{\partial}{\partial B}\frac{1}{\nu_1}\left(\frac{B}{K^{scale}}\right)^{\nu_1}K^{scale} = \frac{\partial}{\partial B}\tau^{statutory}\phi^{int}\left(\rho B + \frac{1}{\nu_1}\left(\frac{B}{K^{scale}}\right)^{\nu_1}K^{scale}\right). \tag{16}$$

Differentiating the above gives

$$\left(\frac{B}{K^{scale}}\right)^{\nu_1-1} = \tau^{statutory}\phi^{int}\left(\rho + \left(\frac{B}{K^{scale}}\right)^{\nu_1-1}\right). \tag{17}$$

And solving for B gives

$$B^* = \left(\frac{\tau^{statutory}\phi^{int}\rho}{1-\tau^{statutory}\phi^{int}}\right)^{\frac{1}{\nu_1-1}}K^{scale}. \tag{18}$$

3.3.3 Calibration

To estimate the parameters ν_1 and ν_2 in the leverage cost function, we look at historical data on business leverage ratios and plug the ratio into (18). Note that ρ is not directly visible in the data, so we make the assumption that it matches the return in our steady-state economy.

In the current model, we take $\nu_1 = 3.5$ following Glover, Gomes, and Yaron (2015) and the leverage ratio $B/Kp_K = 0.32$ from Barro and Furman (2018) and Graham, Leary, and Roberts (2015). Using these values, we derive ν_2 from (18),

$$\nu_2 = \left(\frac{\tau^{statutory}\phi^{int}\rho}{1-\tau^{statutory}\phi^{int}}\right)^{-\frac{1}{\nu_1-1}}\frac{B}{p_K K} \tag{19}$$

These time-invariant values of ν_1 and ν_2 are used for all policy runs under the assumption that the leverage cost function depends only on the leverage ratio.

4 Households

In every period, households are endowed one unit of time that can be allocated to work and leisure. However, at age J_r, households are forced to retire and start receiving Social Security benefits, $ss(b, i)$[5]. Labor supply, n, and savings, a', are continuous choice

[4] Our tax benefit expression assumes the tax shield is maintained in any continuation (e.g., bankruptcy).

[5] The Social Security system is described more fully in a later section. Benefits depend on average lifetime labor earnings, b, and membership in cohort i.

variables. Consumption is taken to be the residual of after-tax income and savings. Households derive utility from consumption, c, and leisure, $1 - n$.

4.1 Labor productivity

Households' labor productivity z has four components:

$$z = z_{age} + z_{perm} + z_{trans} + z_{pers}. \tag{20}$$

When households enter the economy, they draw a permanent component of their labor productivity, z_{perm}. With probability p_{permH}, a household has a high permanent component, and with probability $(1 - p_{permH})$, a low one. There is a deterministic life-cycle component of labor productivity, z_{age}, that varies with age. And, finally, there are two idiosyncratic shocks on households' labor productivity received each period: $z_{trans} \sim N(0, \sigma^2_{trans})$ is a transitory shock and z_{pers} is a persistent shock, which follows a first-order autoregression:

$$z'_{pers} = \rho z_{pers} + \eta'; \quad \eta \sim N(0, \sigma^2_\eta). \tag{21}$$

The following parameter values are used.

Table 1: Labor Productivity Parameter Values

Variable	Description	Value	Source
p_{permH}	High permanent prod. probability	0.500	Storesletten *et al.* (2004)
σ_{perm}	Permanent productivity variance	0.2105	Storesletten *et al.* (2004)
σ_{trans}	Transitory productivity variance	0.063	Storesletten *et al.* (2004)
σ_{pers}	Persistent productivity variance	0.018	Storesletten *et al.* (2004)
ρ	Persistent prod. autocorrelation	0.990	Storesletten *et al.* (2004)

4.2 Death

At each period households aged j survive to the next period with probability s_{j+1} unless $j = J$, in which case they die with probability 1. In the event of death, accidental bequests are collected by the government and uniformly distributed among the living population by means of lump-sum transfers, beq.

4.3 Savings

Households can accumulate positive assets. A unit of asset a is a portfolio that combines a share ψ^{cap}_{port} of capital (in dollars) and a share ψ^{debt}_{port} of government debt, with

$\psi_{port}^{cap} + \psi_{port}^{debt} = 1$. Households take these shares as exogenous. This assumption is required in order to generate a positive demand for both types of assets while imposing a spread between their return rates. Total domestic physical capital in the economy, K_d, equals aggregate savings, A, less government debt held by households, D_d, in units of physical capital (as opposed to dollar units):

$$K_d = \frac{A - D_d}{p_K}. \tag{22}$$

Consequently, we define

$$\psi_{port}^{cap} = \frac{A - D_d}{A}, \text{ and} \tag{23}$$

$$\psi_{port}^{debt} = \frac{D_d}{A}. \tag{24}$$

The above portfolio split applies in every period of the closed economy. Under open economy assumptions, the portfolio allocation is fixed from the economy's initial steady state and held constant along the transition path.

Returns to savings are taken as given by households. Total portfolio return depends on its composition. For physical capital, gross return is MP_K which is endogenous to the economy. However, income delivered to households is filtered through business-level taxation and other adjustments as described in section (3.2) on Production. For government debt, the sequence of government interest rates, r_G, is exogenous to the model and based on projections by PWBM.

Define by $k_{corp} = \zeta_{corp}^{cap} \psi_{port}^{cap} a$ as the portion of a which is held in corporate capital. Note that the units are dollars, not physical capital. Define $k_{pass} = \zeta_{pass}^{cap} \psi_{port}^{cap} a$ as the pass-through portion. For convenience, set $k = k_{corp} + k_{pass}$. Analogously, define $d = \psi_{port}^{debt} a$ as the portion held in government debt.

4.4 Income

There are only two economic sources of income: capital and labor. However, for the purposes of the model's tax system, there are multiple types of income which are derived from these sources:

1. Corporate business sources income
2. Pass-through business income
3. Capital gains
4. Interest on government debt
5. Wages and salaries

6. Social Security benefits

A household's labor income is

$$y_{lab} = wzn, \tag{25}$$

where w is the wage rate which the household takes as given, z is the household specific labor productivity shock, and n is the household's choice of labor supply. Labor sourced income includes wages, benefits, and some income from pass-through business activity.

A household's total income is

$$y_{hh} = y_{lab} + ss(b,i) + \pi_{corp}k_{corp} + \pi_{pass}k_{pass} + r_G d + k\left(p_{K,t} - p_{K,t-1}\right), \tag{26}$$

where household assets may increase at the start of the period due to a change in the cost of capital, p_K. The last term thus defines the change in capital value from period $t-1$ to t. We assume no capital gains are possible on debt.

4.5 Taxation

There are four types of tax treatments in the model at the household level:

1. Ordinary rates treatment (ORD) which operates on wage income, ordinary business income from pass-through business, some corporate distributions, and Social Security income.

2. Preferred rates treatment (PREF) which operates on some corporate distributions.

3. Payroll tax (PT) which operates on wage income and self-employed income.

4. Consumption tax (CON) which operates on household consumption.

All tax functions are time-varying to reflect time-based policy changes and compositional changes where applicable. Each tax function is a cumulative tax liability from a marginal tax rate function, which is a step-function imported from the PWBM TaxCalculator component. It is defined from a set of rates and bracket thresholds between which the rates are applied. The bracket thresholds themselves can be indexed (for instance, to the rate of wage growth in the model). Note that the model itself has no price inflation, so indexing to CPI (i.e., price inflation) is accomplished by holding constant a threshold value. Indexing to a nominal threshold implies deflating the real dollar value by the CPI index. The different tax functions are defined in sections below.

As a convenience, we write $\tau_{HH}(\mathbf{s}, \mathbf{S})$ to describe the cumulative tax liability of a household characterized by state $\mathbf{s}$ at aggregate state $\mathbf{S}$.

4.5.1 Ordinary rates

The ordinary rates (ORD) tax function is a non-linear tax function of household income. Denote this tax function by $\tau_{ORD}(y)$ for household taxable income y [6]. The tax function $\tau_{ORD}(\cdot)$ is a cumulative tax liability from a marginal tax rate function, which is a step-function imported from the PWBM TaxCalculator component. At each segment, this effective marginal tax rate accounts for deductions, credits, and other features not explicitly modeled. The bracket thresholds are indexed to grow with CPI on top of any other bracket threshold shifts provided by the input.

The ORD tax base consists of corporate distributions, pass-through business income, labor income, interest debt income, and Social Security benefits. Corporate income is allocated to ORD and PREF tax treatments according to taxable shares θ^{corp}_{ORD} and θ^{corp}_{PREF}, respectively. Note that these allocations do not need to sum to 100% as some portion of income may be excluded. Income from government debt is allocated in its entirety to ORD. Some labor income is excluded from ORD (for instance, health insurance), so labor income is allocated by θ^{lab}_{ORD}.

Pass-through tax liability is allocated through the adjusted tax-base, Π_{pass}, and allocation adjustment θ^{pass}_{ORD} as

$$\theta^{pass}_{ORD}\Pi_{pass}\frac{k_{pass}}{K_{pass}}. \tag{27}$$

The household's tax liability depends on the amount of pass-through capital owned by the household.

For those households receiving Social Security retirement benefits, a portion of those benefits is allocated to ORD tax treatment by $\theta^{ss}_{ORD}(y_{hh})$ which is a function of household gross income.

The tax liability for ORD tax treatment is:

$$\tau_{ORD}\left(\theta^{corp}_{ORD}\pi_{corp}k_{corp} + \theta^{pass}_{ORD}\Pi_{pass}\frac{k_{pass}}{K_{pass}} + \theta^{lab}_{ORD}y_{lab} + \theta^{ss}_{ORD}(y_{hh})ss(b,i) + r_G d\right).$$

4.5.2 Preferred rates

A share θ^{corp}_{PREF} of corporate income is allocated to preferred rates (PREF) tax treatment. As with the ORD function, the tax function $\tau_{PREF}(y)$ is a cumulative tax liability from a marginal tax rate function, which is a step-function imported from the PWBM TaxCalculator component. The function operates on allocated preferred rate income. The bracket thresholds are indexed to grow with CPI on top of any other bracket threshold shifts provided by the input.

[6] Although the current implementation is based only on household income and not other elements of household state, we plan to expand this function in the near future. For instance, since older households are more likely to have a mortgage, effective tax rates for households of the same income but different ages are likely to differ. In this way, age may proxy for unmodeled aspects of the tax code.

The tax liability for PREF tax treatment is

$$\tau_{PREF}\left(\theta_{PREF}^{corp}\pi_{corp}k_{corp}\right). \tag{28}$$

4.5.3 Payroll tax

Payroll tax (PT) is assessed on wage income. The portion of labor income allocated to PT is θ_{PT}^{lab}. As with the ORD function, the tax function $\tau_{PT}(y)$ is a cumulative tax liability from a marginal tax rate function, which is a step-function imported from the PWBM TaxCalculator component. Under current policy, the bracket thresholds are indexed to grow with the wage index on top of any other bracket threshold shifts provided by the input. Counterfactual policies can choose a different index available in the model.

Although current law provides for a single tax rate up to a taxable maximum, we implement a full tax function to accomodate various policy proposals. Payroll tax includes both the employee and employer portion. In the current version, we model only FICA taxes and do not model Medicare taxes.

Tax liability for PT tax treatment is $\tau_{PT}(\theta_{PT}^{lab}y_{lab})$.

4.5.4 Consumption tax

Consumption taxes are levied on household consumption. In addition, we allow a lump sum tax per household and integrate this tax into the tax burden. Different consumption goods have different elasticities of demand. We can approximate the tax effect of different elasticities by a combination of lump sum tax, which implies inelastic demand, and a consumption tax on all goods, which can approximate some of the elasticity for the particular sub-set of goods which are being taxed. This approach allows us, in an admittedly rough, reduced form way, to model consumption taxes on specific subsets of goods. The consumption tax is given by the function:

$$\tau_{CON}(c) \equiv \tau_{con}c + \tau_{lumpsum}, \tag{29}$$

where τ_{con} is the tax rate imposed on consumption, c is the household's choice of consumption, and $\tau_{lumpsum}$ is the per-household tax amount which is normally set to zero.

4.5.5 Capital gains and estate taxes

We do not tax capital gains and bequests. In the case of bequests, the bequest modeling is too simple to allow for taxation which reflects any actual inheritance/estate taxes. Since estate taxes are small, it seems reasonable to ignore them. In the case of capital gains, note that the model's capital gains result solely from a change in the user cost of capital. Furthermore, realizing capital gains and thus the resultant taxation is a choice by households which we do not model. It is very difficult to determine this component of empirically realized capital gains, so we have not attempted this yet.

Capital income distributions in the model include not only dividends, but also share buybacks and any other methods of delivering income to shareholders. Increases in the value of the asset are possible only from a change in p_K. Thus, capital gains from any capital income distributions are implicitly modeled as part of the empirically measured tax functions $\tau_{ORD}(\cdot)$ and $\tau_{PREF}(\cdot)$.

4.6 Household problem

Recall that a household's type is given by $\mathbf{s} = (j, a, z, b)$, where j denotes age, a denotes assets, z denotes labor productivity and b denotes average lifetime labor earnings used for calculation of Social Security pension benefits. Let $V(\mathbf{s}, \mathbf{S}; \mathbf{\Psi})$ denote the value of type $\mathbf{s}$ households for a given government policy schedule, $\mathbf{\Psi}$, at the beginning of the period.

The household's period utility function is given by:

$$u(c, n) = \frac{(c^{\gamma}(1-n)^{1-\gamma})^{1-\sigma}}{1-\sigma}, \tag{30}$$

where γ is the elasticity of substitution between consumption and leisure, σ is the intertemporal elasticity of substitution, and β is the time preference parameter – all calibrated to match targets of the U.S. economy in 2017. This function increases in consumption and leisure time.

The household Bellman's equation is given by:

$$V(\mathbf{s}, \mathbf{S}; \mathbf{\Psi}) = \max_{c, a', n} \{u(c, n) + s_{j+1}\beta E\left[V(\mathbf{s}', \mathbf{S}'; \mathbf{\Psi}')\right]\} \tag{31}$$

$$\text{s.t.}$$

$$c = y_{hh} + a - a' + beq - \tau_{HH}(\mathbf{s}, \mathbf{S}) \tag{32}$$

$$c, a', n \geq 0, \tag{33}$$

where s_{j+1} is the survival probability to next period, (32) is the budget constraint, c is consumption, n is the choice of current period labor, y_{hh} is total income as defined in equation (26), a is current period assets in dollars, a' is choice of next period assets (purchased this period), beq is bequests received, and τ_{HH} is the total tax liability, given by the sum $\tau_{ORD}(\cdot) + \tau_{PREF}(\cdot) + \tau_{PT}(\cdot) + \tau_{CON}(\cdot)$.

Working age $j < J_r$ households have no pension income, $ss(b, i) = 0$. For these households, we calculate average life-time wage income relevant to Social Security benefit by a function $sswage(\cdot)$, defined in section (7) on Social Security benefits. The state parameter b evolves according to:

$$b' = \frac{1}{j}\left((j-1)b + sswage(y_{lab})\right), \tag{34}$$

where y_{lab} denotes labor income, defined as in equation (25). For retired households, $n = 0$ and so $y_{lab} = 0$. The state parameter b determines the pension benefit $ss(b,i)$ which is cohort specific and also detailed in the section on Social Security. The retired b evolves according to:

$$b' = b. \tag{35}$$

5 Government and its debt

The government collects taxes, spends money, and issues debt to finance deficits. The government's debt (which is calibrated to current debt levels initially) can grow or shrink depending on the mismatch between spending and revenues. In the current version of the model, only retirement benefits exist as a transfer payment. Other government expenditures, though important to calibrate debt, are ignored – "thrown in the ocean" – and have no direct consequence for any agent.

As discussed in subsection (4.3), the market for government debt is forced to clear regardless of price – that is, the interest rate on debt. In the closed economy, households are forced to buy up all debt into their portfolios; this changes the composition of the portfolio between capital and debt. In the open economy, household portfolio allocations are fixed and any debt in excess of that is purchased by households is purchased by foreigners.

5.1 Interest rate on debt

We take the interest rate on government debt as exogenous. The interest rate projections come from the PWBM microsim. While this approach can be faulted for not taking into account equilibrium effects on debt interest rates, the resultant model can be viewed as a conservative projection of debt effects – that is, rising debt and thus interest rates may make things worse than what is projected by the model.

5.2 Expenditures

Of government expenditures, only Social Security retirement benefits are explicitly modeled[7]. We denote total benefit expenditures by $SSBEN$. All other expenditures are calibrated to match the government expenditure share of GDP as projected by the

[7]See section (7) on Social Security for details.

PWBM microsim. This residual is denoted as $\tilde{G}$. Once the vector $\tilde{G}$ is calibrated from the baseline transition path, those values are used for counterfactual experiments as dollar values. These expenditures have no direct consequence to any agent except through the indirect effect on debt.

5.3 Revenues

Define total in-model tax revenues by

$$
\begin{aligned}
T = & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (36) \\
& \int_{\mathbf{s}} \tau_{HH}(\mathbf{s}, \mathbf{S}) x(\mathbf{s}) d\mathbf{s} && \text{(Households)} \\
& + T_{corp} && \text{(Corporates)} \\
& + \tau_{FOR}^{corp} \pi_{corp} p_K K_{f,corp} + \tau_{FOR}^{pass} \pi_{pass} p_K K_{f,pass}. && \text{(Foreigners)}
\end{aligned}
$$

Note that $\tau_{HH}(\cdot)$ includes all household taxation, including payroll tax and consumption tax. Corporate tax revenues, T_{corp}, are defined in subsection 3.2.1, on Production.

While the model has a fairly detailed tax system, including a variety of business entity-level and household-level taxation, various sources of revenue are not modeled (e.g., excise taxes). Therefore, calibrating the model to tax revenue as a share of GDP, necessarily results in a residual. Denote this revenue residual by $\tilde{T}$.

Normally, one should be concerned that money magically appears in the model economy, as with the $\tilde{T}$ term, but the relevant quantity is actually $\tilde{G} - \tilde{T}$. If this quantity is non-negative, then no new money is added. If $\tilde{G}$ throws money in the ocean, then $\tilde{T}$ fishes some of it back out. Since neither residual has any direct utility consequence to the model's agents, no distortion to choices occurs. Matching the debt, on the other hand, calibrates debt's effects on the model. Admittedly, if $\tilde{G} - \tilde{T} < 0$, then magic money appears in the model from unspecified outside sources.

5.4 Closing the debt

On a debt-growth trajectory where debt grows faster than output (as under current policy), the model becomes unsolvable. At some point, debt and debt service absorb the entire economic output of the economy which then shrinks to zero. Agents are forward-looking, so the existence of this doomsday makes current optimal behavior difficult to define.

To resolve this issue, some magic is called for. In the case of an open economy, we make the assumption that foreigners will absorb arbitrarily large amounts of U.S. debt, regardless of whether they can be repaid. For the closed economy, we take the approach that in some future year [8] , the government will fix the debt-to-GDP ratio and maintain that debt-to-GDP forever. In order to do this, we introduce a residual $\tilde{C}$ which is money necessary to reduce the debt to a particular debt-to-GDP ratio which is the debt-to-GDP at the chosen year. The source of $\tilde{C}$ is unspecified. Model integrity is maintained so long as $\tilde{G} - \tilde{T} - \tilde{C} \geq 0$. One can think of the government as reducing expenditures in order to prevent runaway debt. However, past a certain point, expenditure reduction is insufficient. A variety of policies are possible to reduce debt (e.g., some type of default or forebearance), but we do not specify the policy other than through the effect via $\tilde{C}$.

5.5 Debt equation

In accordance to the above, government debt D evolves as:

$$D' = D(1 + r_G) + (\tilde{G} + SSBEN) - (T + \tilde{T} + \tilde{C}). \tag{37}$$

6 Open and closed economies

To start, we discuss international trade and finance through one of two simple models: (a) a totally closed economy and (b) a small open economy. It is clear that these models are unrealistic abstractions. We use the steady-state model to set initial conditions for the economy. This initial economy is considered to be neither open or closed since the economy is on a stationary growth path (with a fixed debt-to-GDP ratio).

Openness of the economy becomes meaningful along the transition path. Foreign flows absorb some U.S. government debt and provide additional physical capital to the U.S. economy.

6.1 Closed economy

In the closed economy model, there are no capital flows to or from the U.S. economy. In equilibrium, given prices $\pi_{corp}, \pi_{pass}, r_G$, and w, markets clear as:

$$\int ax(\mathbf{s})d\mathbf{s} = p_K K + D \qquad \text{(Asset)} \tag{38}$$

$$\int znx(\mathbf{s})d\mathbf{s} = L. \qquad \text{(Labor)} \tag{39}$$

In words, markets clear when households' demand for assets equals government debt plus the firm's demand for capital at the prices in the economy. Similarly, household supply of effective labor matches the firm's demand for labor. Since the goods price is numeraire, the market for goods clears when all production Y is accounted for by

[8] The exact year chosen is at the discretion of the modeler. We usually use 2040.

aggregate consumption, government spending, capital investment, and other spending and losses. Given the complexity of the model, we do not write the full accounting here.

Capital income returns/prices π_{corp} and π_{pass} are determined by the marginal product of capital, MP_K, while the wage rate is determined by the marginal product of labor, $w = MP_L$, under the assumption of perfectly competitive markets. Since the production function is Cobb-Douglas, the capital-labor ratio K/L determines both.

Note that the asset market is dominated by government debt D. That is, capital is a residual after assets have been purchased by households. The level of physical capital in the closed economy is determined by:

$$K = K_d = \frac{1}{p_K}\psi^{cap}_{port}\int ax(\mathbf{s})d\mathbf{s}. \tag{40}$$

This "crowding out" effect reduces productive capital as debt increases.

6.2 Open economy

In the small open economy model, there are international capital flows, implying that foreigners own some of the total physical capital of the economy, that is:

$$K = K_f + K_d, \tag{41}$$

where the term K_d denotes physical capital held by domestic households, as defined in (40), and K_f denotes physical capital held by foreigners. Notice that $K_f > 0$ implies foreigners hold domestic capital, while $K_f < 0$ implies domestic households hold foreign capital abroad in net terms.

Foreign owners of corporate capital pay the tax τ^{corp}_{FOR} which represents taxation on corporate payouts to foreign shareholders. Similarly, foreign owners of pass-through capital pay the tax τ^{pass}_{FOR}.

Under the small open economy assumption, the marginal after-tax return to capital is constant in the international capital market, so $r_{K,world}$ is a time-invariant constant. Thus, a foreign owner of capital expects a marginal return of:

$$r_{K,world} = \frac{\partial}{\partial k}\left(\pi_{corp}k_{corp}p_K(1-\tau^{corp}_{FOR}) + \pi_{pass}k_{pass}p_K(1-\tau^{pass}_{FOR}) + (p_{K,t} - p_{K,t-1})\,k\right), \tag{42}$$

where $k = k_{corp} + k_{pass}$ with the split between the two types of capital determined exogeneously. Since p_K and τ_{FOR} are changed only by policy, under the assumptions of the small open economy, it must be that π_{corp} and π_{pass} adjust. The mechanism for adjustment is the change in capital held by foreigners.

The returns π_{corp} and π_{pass} are affected by MP_K which is determined by the K/L ratio. Foreigners invest for next period just as U.S. households do. Investment generates tax credits (from investment expensing), so current period return is affected by both current period capital and by investment for next period. The solution to the open economy problem involves a vector of foreign capital K_f residuals such that (42) holds in all periods except possibly the first period (where the policy shock and time delay for investment prevent immediate regression to the world rate). Multiple equilibria exist in this solution, but we choose the sensible one with a reasonable ratio of capital to investment. A further discussion of this issue and more details on the solution are in (Berkovich and Costa 2018).

Foreign-held debt D_f is also a residual in the equation

$$D = D_f + \psi_{port}^{debt} \int adX(\mathbf{s}), \tag{43}$$

where the last term is debt held by U.S. households under the assumption that portfolio allocation in the small open economy is exogenously determined.

6.3 Foreign taxes

Foreigners pay corporate taxes implicitly through share ownership. In addition, foreigners pay taxes on dividend distributions and on returns from pass-through capital. These taxes are collectible by the U.S. government since they can be assessed on the U.S. business entity. No taxes on capital gains are collectible by the U.S. government. Although the statutory rate on corporate distributions to foreign shareholders is 30%, a variety of tax treaties reduce this rate. The PWBM estimate of the effective tax rate aggregates across foreign share ownership by country and is about 17% under current policy. The effective tax rate on pass-through income to foreign shareholders is about 8% under current policy.

6.4 Partially open economy

To model an economy where some international flows exist but at a level less than at the fully open economy, we implement (a) debt take-up and (b) capital take-up parameters which control the degree to which foreign financial flows enter the U.S. economy. These time-varying parameters are currently taken as exogenous.

For debt take-up, the parameter $\zeta_{foreign-takeup}^{debt}$ is the amount of new debt which is acquired by foreigners. PWBM estimates that in recent history about 40% of new debt has been taken up by foreign investors. The definitions of foreign and domestically held debt are

$$D_f = (D' - D)\,\zeta^{debt}_{foreign-takeup} \tag{44}$$

$$D_d = D - D_f. \tag{45}$$

Government debt still dominates the asset market for U.S. households just as in the case of the fully closed economy in that U.S. households are forced to clear the market for government debt.

Capital take-up is less straightforward than debt take-up because the total size of the capital market is not defined ex ante, unlike the debt series D. Rather, investment is driven by savings which is driven by the endogenous rate of return on investment. We define the capital take-up parameter $\zeta^{capital}_{foreign-takeup}$ to be a percentage of capital flow under the assumption of a fully open economy, denoted here by K^{open}_f, from (41) so that (42) holds. Thus,

$$K_f = \zeta^{capital}_{foreign-takeup} K^{open}_f, \tag{46}$$

and domestically held capital K_d is defined exactly as for the fully closed economy in (40). Portfolio allocation by U.S. households is defined (as above) by

$$\psi^{cap}_{port} = \frac{A - D_d}{A}, \text{ and} \tag{47}$$

$$\psi^{debt}_{port} = \frac{D_d}{A}, \tag{48}$$

where domestically held government debt forces domestically held capital to be a residual in the portfolio.

Foreign take-up of debt reduces capital "crowd out" while foreign capital changes the total capital stock and thus affects prices in the economy. In this way, a partially open economy reacts less than a fully closed economy to the growth of government debt. The partially open economy, unlike the small open economy, can collapse if debt grows too quickly.

7 Social Security Benefits

The calculation of a household's Social Security benefits is a complex one. Our model does the best to resemble the actual policy while making key assumptions that keep it treatable. Since households in our model do not get sick or disabled and are not grouped as a family, the first simplifying assumption is that all Social Security benefits refer to retirement benefits. In this section, we briefly describe how the Social Security Administration (SSA) computes retirement benefits and how we translate that into our model. Note that the values provided by the SSA and our projections are in nominal terms, so we bring them to real terms before using them as parameters in our model.

7.1 Normal retirement age

Following the increasing life expectancy and the improvements in the health of older people, the average retirement age has been increasing. Based on that, the SSA sets a normal (or full) retirement age (NRA) for each cohort, born in a given year, used to set a number of policies. For instance, the earliest a person can start receiving Social Security retirement benefits is at age 62. However, if one starts receiving benefits at that age, one will get only 70 percent of the monthly benefit. As one postpones the decision to retire, this percentage increases, reaching 100 percent at the NRA and going above 100 percent in case of retirement beyond the NRA.

In our model, we impose that a household must retire at its cohort's NRA. In other words, we do not internalize the retirement timing decision and how it interacts with the benefits received. Doing so would add an extra touch of realism to our model, but at a high cost, since it would also significantly increase the state space, and with low gain, since on average cohorts retire at their NRA, impacting little economic aggregates[9]. We then opt for the more parsimonious approach.

7.2 Average indexed monthly earnings

In the economy and in the model alike, the history of one's labor earnings determines the amount received as retirement benefits. This history is condensed into one variable that the SSA denotes the average indexed monthly earnings (AIME) and that we denote b in our model.

The first step to calculate the AIME is to index nominal earnings, converting past earnings to approximately their equivalent values near the time of the person's retirement. The index used in this step is known as the national average wage index and is the result of dividing the average wage index for the year in which the person attains age 60 by the average wage index of the current year. After a person attains age 60, this indexing factor remains fixed at 1. The next step is to sum the highest 35 years of indexed earnings and divide it by the number of months in 35 years. The result is the AIME.

Our model performs a similar indexation of earnings, seeing that it is possible that labor is more or less productive on average in one year than in another. In contrast, the model is unable to generate the full historic series of wages nor its future projections, hence for the cohorts with missing indices for some years, we use the steady-state value. Another difference between model and the actual AIME calculation is that we use average

[9] Though aggregates would not change much, allowing households to decide when to retire may accentuate wealth inequality in the model by stimulating the most productive workers to remain in the labor force longer.

earnings across all the years because otherwise, we would have a much larger state space. Before summing all indexed earnings, we cap them to a ceiling, known as the contribution and benefit base (or taxable maximum)[10]. The SSA performs this capping in a future step, but for convenience, we do it at this point. Finally, since a period in the model corresponds to one year, we divide such a sum by the number of active/working years of that person (determined by her NRA).

In short, the model stores the labor earnings information of a household characterized by (j, a, z, b) as follows:

$$b' = \frac{1}{j}\left[(j-1)b + \iota^{wage}_{j=60} min\left\{wzn, \bar{b}\right\}\right], \tag{49}$$

where $\bar{b}$ is the annual labor earnings ceiling and the wage index is defined as:

$$\iota^{wage}_{j=60} = \begin{cases} \frac{w_{j=60}L_{j=60}/\int n_{j=60}(s)x(s)ds}{wL/\int n(s)x(s)ds} & \text{if } j \leq 60 \\ 1 & \text{otherwise.} \end{cases} \tag{50}$$

That is, average wages when household is 60 divided by average current wages[11].

7.3 Primary insurance amount

Finally, the primary insurance amount (PIA) is the benefit (before rounding down to next lower whole dollar) a person would receive if she elects to begin receiving retirement benefits at her NRA. The PIA is the sum of three separate percentages of portions of AIME. The portions depend on the year in which a worker attains age 62 and are known as bend points. The percentages are 90, 32, and 15 percent. The last percentage applies to the top portion of the AIME, capped by the contribution and benefit base.

Our model uses the cohort-specific bend points to create brackets for b (already capped in the previous step) over which the respective percentages apply. For each cohort i, there are two brackets, with thresholds represented by $b_{1,i}$ and $b_{2,i}$. The result is the following retirement benefit:

$$ss(b,i) = \begin{cases} 0.9b & \text{if } b \leq b_{1,i} \\ 0.9b_{1,i} + 0.32(b - b_{1,i}) & \text{if } b_{1,i} < b \leq b_{2,i} \\ 0.9b_{1,i} + 0.32(b_{2,i} - b_{1,i}) + 0.15(b - b_{2,i}) & \text{if } b > b_{2,i}. \end{cases} \tag{51}$$

[10]Social Security's Old-Age, Survivors, and Disability Insurance (OASDI) program limits the earnings used in the computation of benefits for a given year. The same annual limit also applies to earnings subject to payroll taxation.

[11]Notice that $sswage(y_{lab})$ from equation (34) corresponds to:

$$sswage(y_{lab}) = \iota^{wage}_{j=60} min\left\{y_{lab}, \bar{b}\right\}.$$

According to the SSA (11), about 40 percent of people who get Social Security have to pay income taxes on their benefits. No one pays federal income tax on more than 85 percent of his or her Social Security benefits based on Internal Revenue Service (IRS) rules. In order to capture that, our model establishes that 15 percent of the benefit is not subject to income taxes.

7.4 Payroll tax

The payroll tax function $\tau_{PT}(\cdot)$ has already been presented in the Taxation subsection (4.5.3). Nevertheless, it is worth to remind that we model only FICA taxes, as our model does not have a health sector. It is assessed on wage income, up to the contribution and benefit base (or taxable maximum) at a flat FICA rate.

8 Recursive competitive equilibrium

Let $\mathbf{s}_t = (j, a, z, b)$ be the individual state of households, let $\mathbf{S}_t = (x_t(\mathbf{s}), K_t, D_t)$ be the aggregate state of the economy, and let $\mathbf{\Psi}$ be the government policy schedule committed at the beginning of period $t = 1$ and going on forever. The schedule $\mathbf{\Psi}$ includes all tax policies, Social Security benefits policies, expenditure policies, other revenue policies, bequest distribution policies, and so on. Given the complexity of the model, we do not specify the components of $\mathbf{\Psi}$ here.

A recursive competitive equilibrium consists in a time series of factor prices and government policy variables,

$$\mathbf{\Omega} = \left\{ w_t, \pi_{corp,t}, \pi_{pass,t}, r^{debt}_{corp,t}, r^{debt}_{pass,t}, \mathbf{\Psi} \right\}_{t=0}^{\infty}, \tag{52}$$

the value function of households, $\{V(\mathbf{s}_t, \mathbf{S}_t; \mathbf{\Psi})\}_{t=0}^{\infty}$, the decision rules of households,

$$\left\{ c(\mathbf{s}_t, \mathbf{S}_t; \mathbf{\Psi}), n(\mathbf{s}_t, \mathbf{S}_t; \mathbf{\Psi}), a'(\mathbf{s}_t, \mathbf{S}_t; \mathbf{\Psi}) \right\}_{t=0}^{\infty} \tag{53}$$

and the distribution of households, $\{x_t(\mathbf{s_t})\}_{t=0}^{\infty}$, such that, for all t, each household solves the optimization problem (31)-(33), taking $\mathbf{S}_t$ and $\mathbf{\Omega}$ as given; the firm solves its profit maximization problem (14); the government policy schedule satisfies conditions (36)-(37), and the factor markets are cleared as shown in Equations (38)–(39). The economy is in a steady-state equilibrium, and thus on a balanced-growth path, if, in addition, $\mathbf{S}_{t+1} = \mathbf{S}_t$ for all t and the government policy schedule is time-invariant.

In the above competitive equilibrium, the resource (feasibility) constraint is satisfied – the goods market clears – in each period by Walras' law.

9 Generating model output

The OLG GE model has a relatively small state space, as compared to the set of available economic variables, because of computational limitations. The PWBM microsim has a larger state space, but this reduced-form model's projections do not account for changes in a structural way. Because of concern that reduced-form models may not correctly describe real-world behavior when conditions deviate from historical data, the structural approach is preferred for estimating effects from policy changes. On the other hand, the smaller dimensionality of the structural model reduces its precision in forecasting the baseline.

For these reasons, we combine results from the two models in order to set levels from the larger dimensional microsim and calculate deviations from baseline using the structural OLG GE model. We claim that these hybrid results are a better estimate than either model individually.

In the OLG model, define a "static" model run to be one where the agents' policy functions are fixed to the policy functions of the baseline. The "static" model should match as much as possible the assumptions of the reduced-form model used for the PWBM microsim projection.

In this way, the ratio

$$\Delta = \frac{\text{dynamic counterfactual}}{\text{static counterfactual}} \tag{54}$$

should capture just behavioral and general equilibrium effects net of the baseline. This percentage difference between the two model runs is then applied to the projection series from the microsim. The resulting generated series combines the two models under the assumption that the Δ effects are orthogonal to any effects projected in the microsim. For instance, the OLG model does not endogenously model income shifting between pass-through and corporate businesses, but the microsim does provide that modeling (in a reduced-form way). Since the Δ does not account for income shifting, this combination is consistent. As a counter-example, it would not be consistent for the microsim to model consumption changes between baseline and counterfactual because consumption choice is modeled in the OLG model and so applying the Δ would be double counting. This Δ approach does not produce entirely consistent projections because these hybrid results do not account for any potential interaction between income shifting and choices of households for labor supply and saving. Nonetheless, the hybrid results incorporate the direct reduced-form effect of income shifting plus dynamic responses (which ignore income shifting).

The generated series for a particular counterfactual are defined as in this example for GDP:

$$\text{GDP} = \Delta_{\text{GDP}} \cdot \text{GDP}_{\text{microsim}}. \tag{55}$$

Note that Δ is 1 for current policy by definition — that is, there are no deviations from baseline.

Additional details of the approach are available on separate documentation on the PWBM website.

10 Conclusion

The PWBM Dynamic OLG model is a general equilibrium, overlapping generations, incomplete markets model which is computed through backwards value function iteration. Households in the model are distinguished by age, idiosyncratic labor productivity shocks, asset holdings and earnings history. Two economic pricing scenarios are allowed: (a) a "closed" economy where wages and return to physical capital investment are computed by iterating over the capital-labor ratio towards a fixed point and (b) an "open" economy where the after-tax return to capital is fixed and wages are derived from the resulting capital-labor ratio.

The model obtains inputs from PWBMsim to drive the dynamic model's robust taxation system, demographics and immigration flows and Social Security system. Government expenditures and tax revenues from these components generate changes in government debt. This debt feeds back into the model as an asset which can crowd out investment into productive capital in the model economy.

Because of the large set of policy features and the calibration of the model from rich demographics projections of PWBMsim, the PWBM Dynamic OLG model produces policy experiment projections which (a) take account of interactions between different areas (e.g., immigration's effect on Social Security) and (b) provide a match to detailed demographics and policy nuances which are ordinarily not explicitly available in a dynamic model due to state-space limitation. These features make the Penn Wharton Budget Model a best-in-class economic model and give policymakers and researchers a high degree of confidence in using it for projections and analysis.

11 References

Barro, J. and S. Barnes (2016). "Federal Subsidization and State Medicaid Provision," *Review of Economic Dynamics*, 21, 29–45.

Barro, R., and J. Furman (2018). "The macroeconomic effects of the 2017 tax reform," *Brookings papers on economic activity*

Berkovich, E. and D. Costa (2018). "Investment expensing in a small open economy," *Working Paper*, Penn Wharton Budget Model.

Castaneda, A., J. Diaz-Gimenez, and J. Rios-Rull (2003). "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111(4), 818–857.

Conesa, J. C., D. Costa, P. Kamali, T. J. Kehoe, V. Nygard, G. Raveendranathan, and A. Saxena (2017). “Macroeconomic effects of Medicare,” *The Journal of the Economics of Ageing*, forthcoming.

De Nardi, M. (2015). “Quantitative Models of Wealth Inequality: A Survey,” NBER Working Papers 21106, National Bureau of Economic Research, Inc.

Glover, B., J. F. Gomes, and A. Yaron (2015). “Corporate Taxes, Leverage, and Business Cycles,” *Working paper*.

Graham, J. R., M. T. Leary, and M. R. Roberts (2015). “A century of capital structure: The leveraging of corporate America,” *Journal of Financial Economics*, 118(3), 658–683.

Nishiyama, S. and K. Smetters (2013). “Analyzing Fiscal Policies in a Heterogeneous-Agent Overlapping-Generations Model,” Kenneth L. Judd and Karl Schmedders, eds., *Handbook of Computational Economics*, Volume 3, Chapter 3.

Social Security Administration (2018). “Retirement Benefits: Securing Today and Tomorrow,” available at https://www.ssa.gov/pubs/EN-05-10035.pdf.

Storesletten, K., C. Telmer and A. Yaron (2004). “Consumption and Risk Sharing Over the Life Cycle,” *Journal of Monetary Economics*, 51, 609–633.