Economic Projections

Aggregate Output, Prices, Income, and Interest Rates

Penn Wharton Budget Model

This Version: 2026-11-11

Abstract

The economic projections model translates externally supplied aggregate economic paths into a consistent set of annual inputs for budget and policy analysis. Historical national accounts anchor the levels of output and income. Growth in business and nonprofit activity drives total and business output, while a changing nominal sector share governs the translation into real gross domestic product. Price indexes preserve historical inflation differentials, and separate timing rules produce fiscal-year and tax-indexing measures. Historical sector shares allocate business activity across legal forms and income categories. A term-structure model combines Treasury yields, Federal Open Market Committee projections, and trend real growth to project interest rates; a fitted income-share equation then projects monetary interest received by persons. This account states the supplied inputs, calibration rules, economic closure, and horizon conventions needed to reconstruct these quantities.

Contents

1 Economic purpose and closure **2**

1.1 Inputs, responses, and scenario comparisons . . . 2

1.2 Time, units, and supplied information . . . 3

2 Output and the private-sector growth link **4**

2.1 Nominal total and business output . . . 4

2.2 Private-sector share . . . 4

2.3 Real output with changing sector composition . . . 5

2.4 GDP prices and fiscal-year output . . . 5

3 Consumer prices and indexing periods **6**

3.1 Historical measures and inflation differentials . . . 6

3.2 Fiscal-year consumer price indexes . . . 7

3.3 Tax-indexing windows and observation vintages . . . 7

4 Labor aggregates and business accounts **8**

4.1 Employment, population, wages, and productivity . . . 8

4.2 Business output by legal form . . . 9

4.3 Income and investment allocation . . . 9

5 Interest rates and monetary interest income **10**

5.1 Observed rates and the policy-rate path . . . 10

5.2 Cross-sectional yield curves . . . 11

5.3 Factor dynamics and forecast timing . . . 11

5.4 Monetary interest received by persons . . . 12

5.5 Reported quantities and horizons . . . 13

A Implementation components **14**

A.1 Assembly, inputs and aggregate outputs . . . 14

A.2 Output, prices and labor . . . 14

A.3 Business accounts and interest rates . . . 14

B Selection of the factor dynamics **15**

1 Economic purpose and closure

The Penn Wharton Budget Model (PWBM) uses economic projections to translate aggregate activity into the output, prices, income bases, and interest rates needed for policy and budget calculations. The unit of analysis is an annual national aggregate. The model receives private-sector output, wages, productivity, employment, and population paths from the PWBM Microsim. It combines those paths with national accounts and labor-market history, a supplied business-account benchmark, and financial-market and central-bank data.

The central question is how a given path of private economic activity maps into the broader economic quantities used to calculate taxes, spending, and debt. *Private* in the supplied output paths means business plus nonprofit institutions serving households. General government and the household sector, including owner-occupied housing services, lie outside this concept. Total gross domestic product (GDP) includes those sectors. Business GDP follows the national-accounts sector boundary, which includes government enterprises; it is neither a purely private-ownership measure nor the sum of firms' taxable incomes [1].

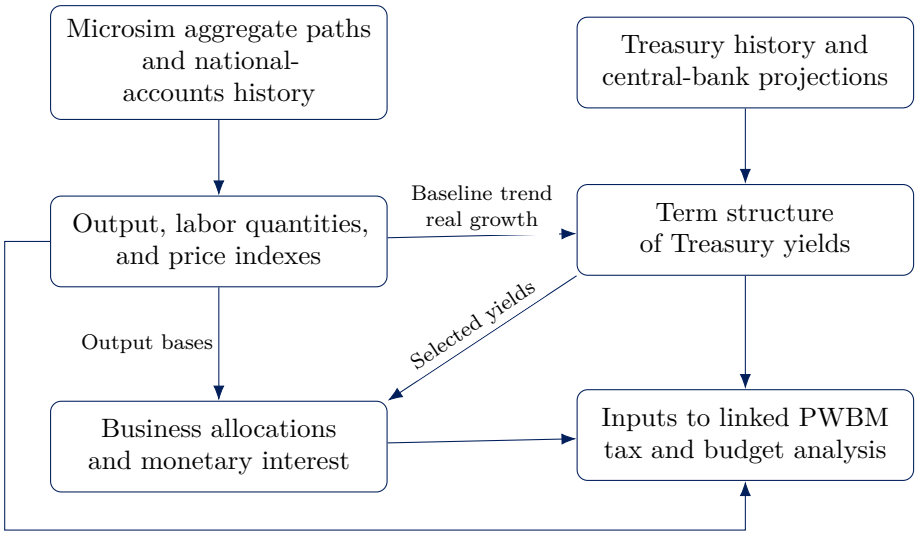

Figure 1: Economic dependencies. A supplied business benchmark determines allocation shares in the business-account block. The baseline growth link supplies a common interest-rate environment.

1.1 Inputs, responses, and scenario comparisons

The economic closure is a set of calibrated aggregate projection rules. The Microsim paths supply the changes in private activity; historical growth ratios translate them into nominal total and business GDP. Real total GDP additionally responds to the lagged nominal private-sector share. The ratio of nominal to real GDP determines the GDP deflator. Other price measures inherit its inflation path plus historical differentials.

Sector income and investment respond proportionally to their assigned output bases. Treasury rates depend on one designated scenario's trend real GDP growth and a central-bank interest-rate path. The designated scenario is the baseline in the selected specification. Every scenario receives that same Treasury yield path. Monetary interest received by persons depends on those yields and each scenario's nominal GDP. These relationships close within the projection system without a

market-clearing interest rate or feedback from projected yields to the supplied Microsim paths.

For a comparison using common historical and business inputs, the calibration windows, sector coefficients, price differentials, and fitted yield relationships remain common. Changing the supplied Microsim paths changes the projected levels and growth rates. Annual growth changes compound into subsequent levels. Switching the designated growth scenario changes the interest-rate environment for all scenarios together. If a scenario is supplied a different business benchmark, its allocation coefficients are recomputed from that benchmark.

1.2 Time, units, and supplied information

Let t denote calendar year, L the last projection year, b the price-index base year, and h_X the last available historical year of quantity X. Different series can have different h_X. Historical values retain their supplied observation years; projection begins at $h_X + 1$. Annual observations can incorporate the producer's available data for an incomplete current year. Supplied series must identify their vintage and coverage.

Symbol	Meaning
N_t, B_t, U_t	Nominal total GDP, business GDP, and nonprofit GDP.
Q_t, Q_t^B, Q_t^U	Real total, business, and nonprofit GDP, in the producer's chained-dollar units.
V_t	Nominal private GDP: business plus nonprofit activity.
X_t^{M}	A supplied Microsim aggregate for economic quantity X.
X_t^{L}	A supplied business-account benchmark series.
$g_t(X)$	Annual fractional growth, $X_t/X_{t-1} - 1$.
$\langle x_t \rangle_{\mathcal{W}}$	Arithmetic mean of available observations over calibration years $\mathcal{W}$.
D_t, P_t^j	GDP deflator and price index j, normalized to one in year b.
$F_t, y_t(\tau)$	Federal funds rate and Treasury yield at maturity τ years, in percent per year.
M_t	Monetary interest received by persons, in billions of nominal dollars.

Table 1: Recurring notation; a superscript H denotes observed history.

Output and income flows are in billions of nominal dollars. Real output retains the national accounts' chained-dollar reference year; rebasing a price index to b does not change that real-output reference year. Employment is in thousands of employed persons. Population is in millions of persons. Productivity and ordinary price indexes are dimensionless, with value one in b. Tax-indexing measures retain published index units. Interest rates use percentage points: a recorded value of 4 means 4 percent, whereas growth $g_t(X) = 0.04$ means 4 percent.

The empirical inputs consist of the complete historical series needed for these windows and their preceding years, the Microsim paths through their last supplied year, business benchmark ratios, and the financial series described below. Historical dollar and population series are converted into the output units before use. Missing cells are unavailable observations, never zero economic amounts. Growth calculations require adjacent annual levels. Averages omit missing observations under the rules specified for each calibration; an undefined calibration coefficient does not define an economic projection.

Assumption or calibration choice	Selected value
Projection endpoint L; index base year b	2176; 2023
Output growth calibration $\mathcal{W}_G$	2000–2025, inclusive
Business allocation calibration $\mathcal{W}_B$	2000–2019, inclusive
Price-gap calibration $\mathcal{W}_P$	2015–2019, inclusive
Monetary-interest estimation start	1990
Yield-factor estimation span	Available joint annual history
Trend-growth window; sensitivity θ	Up to ten forward annual rates; 0.5

Table 2: Dated model assumptions for this version. These are calibration and projection choices rather than statutory parameters.

2 Output and the private-sector growth link

2.1 Nominal total and business output

The Bureau of Economic Analysis (BEA) supplies total nominal GDP from National Income and Product Accounts (NIPA) table 1.1.5 and business and nonprofit GDP from table 1.3.5. Define historical private output as

$$V_t^{\mathrm{H}} = B_t^{\mathrm{H}} + U_t^{\mathrm{H}}. \tag{1}$$

The model translates supplied nominal private growth into each broader output concept by calibrating an average ratio of annual growth rates. To specify the treatment of zero growth, define

$$R(a,c) = \begin{cases} a/c, & c \neq 0, \\ 1, & a = c = 0, \\ \text{omitted from the average}, & c = 0,\ a \neq 0. \end{cases} \tag{2}$$

Missing ratios are also omitted. This is an average of annual ratios, with equal weight on retained years, rather than a ratio of multiyear growth rates. For $X = N$ (total GDP) or $X = B$ (business GDP), set

$$k_X = \left\langle R\Big(g_t(X^{\mathrm{H}}), g_t(V^{\mathrm{H}})\Big)\right\rangle_{\mathcal{W}_G}, \qquad X_t = X_{t-1}\left[1 + k_X g_t(V^{\mathrm{M}})\right], \quad t > h_X. \tag{3}$$

The historical level $X_{h_X}^{\mathrm{H}}$ anchors the recurrence. The two output concepts have separate k_X coefficients and historical anchors. A change in private-sector growth therefore affects both total and business GDP, with magnitudes determined by their historical growth ratios. When the Microsim path ends at T_{M}, its final annual nominal private growth rate is held constant through L in equation (3).

2.2 Private-sector share

Real output uses a separate nominal private-output path that preserves the supplied private growth exactly. With h_V the last observed private-output year,

$$V_t = \begin{cases} V_t^{\mathrm{H}}, & t \le h_V, \\ V_t^{\mathrm{M}}\, V_{h_V}^{\mathrm{H}} / V_{h_V}^{\mathrm{M}}, & h_V < t \le T_{\mathrm{M}}, \\ V_{t-1}(1+\bar{g}_V), & t > T_{\mathrm{M}}, \end{cases} \qquad \bar{g}_V = \frac{1}{5}\sum_{j=T_{\mathrm{M}}-4}^{T_{\mathrm{M}}} g_j(V^{\mathrm{M}}). \tag{4}$$

The last historical level determines the scaling factor. The supplied series must cover that anchor and the first projection year, and have at least six annual levels to support the five growth rates. Its historical anchor and its tail rule differ from the growth-ratio rule for total GDP. Define the private-sector share of nominal GDP as

$$s_t = V_t/N_t. \tag{5}$$

Historical private and total GDP supply this ratio throughout the calibration window. Projected values supply it subsequently. Thus the share evolves with the relative growth of the scaled private path and total nominal GDP.

2.3 Real output with changing sector composition

BEA tables 1.1.6 and 1.3.6 supply real total, business, and nonprofit output. Chained-dollar component levels are generally nonadditive [2]. The model therefore constructs the historical real-growth pattern for private activity as a Laspeyres approximation with lagged nominal weights:

$$\gamma_t = \frac{B^{\mathrm{H}}_{t-1}}{V^{\mathrm{H}}_{t-1}} g_t(Q^{B,\mathrm{H}}) + \frac{U^{\mathrm{H}}_{t-1}}{V^{\mathrm{H}}_{t-1}} g_t(Q^{U,\mathrm{H}}). \tag{6}$$

This is the model's aggregation rule for growth, rather than a claim to reproduce BEA's full chain-type index calculation.

Calibrate the historical relative-growth multiplier and average lagged private share:

$$k_Q = \Big\langle R\Big(g_t(Q^{\mathrm{H}}), \gamma_t\Big)\Big\rangle_{\mathcal{W}_G}, \qquad \bar{s} = \langle s_{t-1}\rangle_{\mathcal{W}_G}, \qquad r = \frac{k_Q - \bar{s}}{1-\bar{s}}. \tag{7}$$

Here r is the implied relative-growth contribution of activity outside the private sector. It is calibrated from the aggregate relationship; it is not an independently projected government or household output series. The denominator requires $\bar{s} \neq 1$.

For years beyond observed real GDP,

$$Q_t = Q_{t-1}\Big\{1 + [s_{t-1} + (1-s_{t-1})r]\, g_t(Q^{\mathrm{M}})\Big\}, \qquad t > h_Q, \tag{8}$$

where Q^{M} is the supplied real private-output path. Its final annual growth rate is held constant after its last supplied year. The varying multiplier equals k_Q when the private share equals its historical mean. As the private share rises toward one from below, the multiplier moves toward one. The projection formula itself does not cap the share at one. The derivative of the multiplier with respect to that share is $1 - r$, so its direction depends on the calibrated r. Equations (4)–(8) retain the effect of projected nominal sector composition even after real private growth becomes constant.

2.4 GDP prices and fiscal-year output

The GDP deflator is the normalized implicit price of aggregate output:

$$D_t = \frac{N_t/Q_t}{N_b/Q_b}. \tag{9}$$

Historical nominal and real GDP are used before the projected series' starting span. This identity makes price growth a residual of nominal and real output growth:

$$1 + g_t(D) = \frac{1 + g_t(N)}{1 + g_t(Q)}. \tag{10}$$

Consequently, a scenario can change projected inflation through the differing nominal and real output rules.

Federal fiscal-year GDP retains the supplied historical fiscal-year series from the U.S. Office of Management and Budget, distributed as FYGDP [6]. Beyond its last historical year h_{FY}, calendar-year growth is blended according to the federal fiscal year's October–September timing:

$$g_t^{\mathrm{FY},N} = \tfrac{3}{4} g_t(N) + \tfrac{1}{4} g_{t-1}(N), \qquad N_t^{\mathrm{FY}} = N_{t-1}^{\mathrm{FY}}(1 + g_t^{\mathrm{FY},N}), \quad t > h_{\mathrm{FY}}. \tag{11}$$

This is a growth-rate approximation anchored to fiscal-year history. It is not an arithmetic average of the two calendar-year GDP levels.

3 Consumer prices and indexing periods

3.1 Historical measures and inflation differentials

The personal consumption expenditures (PCE) price index is supplied by BEA's NIPA table 1.1.4 [3]. Consumer price series come from the Bureau of Labor Statistics (BLS): the Consumer Price Index for All Urban Consumers (CPI-U), the index for Urban Wage Earners and Clerical Workers (CPI-W), the Chained Consumer Price Index for All Urban Consumers (C-CPI-U), and the research index for older Americans (R-CPI-E). The latter uses expenditure weights for urban households with a reference person or spouse aged 62 or older. Chained CPI incorporates substitution across spending categories and is subject to revision [7, 8].

The annual CPI-U path uses seasonally adjusted observations. A separate unadjusted CPI-U history supplies fiscal-year and statutory-window measures. The other CPI series use the supplied unadjusted observations. Annual price observations are averages of the available higher-frequency observations distributed by the Federal Reserve Bank of St. Louis through FRED. Each ordinary annual index is divided by its value in year b.

Let P^j be an index being projected and $P^{p(j)}$ its parent. The model uses the following economic dependencies:

Index j	Parent $p(j)$
PCE price index	GDP deflator
CPI-U	PCE price index
CPI-W, R-CPI-E, C-CPI-U	CPI-U

For each link, define the historical inflation differential

$$\delta_j = \left\langle g_t(P^{j,\mathrm{H}}) \right\rangle_{\mathcal{W}_P} - \left\langle g_t(P^{p(j)}) \right\rangle_{\mathcal{W}_P}. \tag{12}$$

The two means separately omit unavailable values. With complete calibration data, this equals the mean annual difference. The historical price level is retained through h_j, and the projection follows

$$P_t^j = P_{t-1}^j \left[1 + g_t(P^{p(j)}) + \delta_j\right], \qquad t > h_j. \tag{13}$$

The parent uses its own historical values and then its projected path. The full parent path extends to L; if a supplied parent growth path ends earlier, its final rate is repeated. The calibration differential is additive in fractional growth rates, so $\delta_j = 0.002$ means a 0.2 percentage-point inflation differential. Downstream differentials accumulate: after both measures enter projection, CPI-U inflation equals GDP-deflator inflation plus the PCE–GDP and CPI-U–PCE differentials. Relative price levels then compound those inflation differences.

3.2 Fiscal-year consumer price indexes

For a calendar-year inflation series π_t, define the fiscal-year growth approximation

$$\pi_t^{\mathrm{FY}} = \tfrac{3}{4}\pi_t + \tfrac{1}{4}\pi_{t-1}. \tag{14}$$

The chained fiscal-year index uses historical and projected C-CPI-U inflation. The unadjusted CPI-U fiscal-year index uses unadjusted historical CPI-U inflation wherever available, then the projected annual CPI-U growth path. Joining growth rates avoids a level discontinuity between the two price-index bases.

Both fiscal-year indexes are fixed to one in b. Their entire histories and projections are constructed by chaining:

$$P_b^{\mathrm{FY}} = 1, \qquad P_t^{\mathrm{FY}} = P_{t-1}^{\mathrm{FY}}(1 + \pi_t^{\mathrm{FY}}) \ (t > b), \qquad P_t^{\mathrm{FY}} = \frac{P_{t+1}^{\mathrm{FY}}}{1 + \pi_{t+1}^{\mathrm{FY}}} \ (t < b). \tag{15}$$

The unadjusted CPI-U fiscal-year series begins in 1987; the chained series begins in 2001. The 75–25 blend approximates the growth of an October–September fiscal-year index from annual information. It does not replace the separate monthly averaging window used for income-tax indexing.

3.3 Tax-indexing windows and observation vintages

Federal income-tax indexing uses a twelve-month window ending in August, as defined in 26 U.S.C. §1(f). The chained measure uses the monthly values available when the initial August index for the preceding tax year is released [11]. This is a common publication vintage for all months in the window.

Let $\mathcal{M}_t$ be the months September of $t-1$ through August of t, and $I_m^{j,v}$ the published monthly index for month m at observation vintage v. The historical window measure is

$$A_t^{j,v} = \frac{1}{|\mathcal{M}_t^{\mathrm{obs}}|} \sum_{m \in \mathcal{M}_t^{\mathrm{obs}}} I_m^{j,v}, \tag{16}$$

where $\mathcal{M}_t^{\mathrm{obs}}$ is the set of admissibly available months. A complete twelve-month window is required, except that the absent October 2025 observation is omitted from its window. Thus the window ending August 2026 averages eleven months under the model's explicit missing-month convention. BLS documents the absence of October 2025 all-items chained CPI [10]. Other incomplete windows remain unavailable.

Three annual measures are supplied:

- The chained window at its own August-release vintage, retrieved as of September 30 of window year t, or the data-retrieval date if earlier. Windows before 2011 use latest available observations because the vintage history begins in 2011.

- The chained window using the latest available monthly observations. Mature base-year windows contain their final revisions.
- The unadjusted CPI-U window using latest available monthly observations, beginning with the window ending in 1947. Both chained measures begin with the window ending in 2001.

The label t is the ending year of the averaging window. A tax adjustment for year $t+1$ uses the newest window ending in t together with the relevant statutory base. Tax-law bases, adjustment ratios, rounding, and dollar amounts are calculated by the receiving tax model. The economic projections supply the price measures.

Historical window averages remain in the published index units. For each measure, projection starts after its own final available window h_A and uses

$$A_t = A_{t-1}\left[1 + \tfrac{2}{3}\pi_t + \tfrac{1}{3}\pi_{t-1}\right], \qquad t > h_A, \tag{17}$$

where π_t is the annual inflation path of the corresponding CPI-U or C-CPI-U measure. Eight months of the window fall in t and four in $t-1$. This blend is the model's annual-data approximation to future window growth. The two chained window series use the same projected inflation rule but retain their own final historical levels and vintage conventions.

4 Labor aggregates and business accounts

4.1 Employment, population, wages, and productivity

Historical employment is the BLS Current Population Survey's count of employed persons in the civilian noninstitutional population aged 16 or older. It counts a person once even if that person has multiple jobs [9]. The historical population input is BEA's population concept used in personal-income accounting: U.S. resident population plus armed forces overseas, including institutional residents [4]. The different universes are retained. Employment and population are separately anchored national totals, rather than an internally calculated employment rate.

For either employment or population, let Z_t^{H} be observed annual history and Z_t^{M} the supplied Microsim path. With T_Z its last supplied year, the projection is

$$Z_t = \begin{cases} Z_t^{\mathrm{H}}, & t \le h_Z, \\ Z_t^{\mathrm{M}} \dfrac{Z_{h_Z}^{\mathrm{H}}}{Z_{h_Z}^{\mathrm{M}}}, & h_Z < t \le T_Z, \\ Z_{t-1}(1+\bar{g}_Z), & t > T_Z, \end{cases} \qquad \bar{g}_Z = \frac{1}{5}\sum_{j=T_Z-4}^{T_Z} g_j(Z^{\mathrm{M}}). \tag{18}$$

The final historical year supplies the national scale, and the final five Microsim growth rates supply the terminal growth rate. Historical values must cover the requested historical output years. The Microsim series must include the scaling year, the first projection year, and enough adjacent annual levels to form those five rates. Multiplying employment by one thousand yields persons; multiplying population by one million yields persons. Even after this conversion, their ratio has a broader population denominator than the BLS employment-to-population ratio.

Total wages and salaries are anchored to BEA NIPA table 1.12. Let W_t^{H} denote that nominal flow in billions and W_t^{M} the supplied aggregate. For $h_W < t \le T_W$,

$$W_t = W_{t-1}[1 + g_t(W^{\mathrm{M}})]. \tag{19}$$

After T_W, use the arithmetic mean of the final five supplied annual wage-growth rates. Thus wages, employment, and population inherit the Microsim's aggregate changes while the observed series establish their national levels. No additional person weights, household adjustments, or wage distribution are constructed here.

Let A_t^{M} denote supplied multifactor productivity. The output index is $A_t = A_t^{\mathrm{M}}/A_b^{\mathrm{M}}$ over its supplied span. Beyond its last year T_A,

$$A_t = A_{t-1}[1 + g_{T_A}(A^{\mathrm{M}})]. \tag{20}$$

This terminal rule uses the last single productivity growth rate. Productivity is a supplied aggregate index in this model; the output equations above already consume the Microsim output paths.

4.2 Business output by legal form

The supplied business-account benchmark divides business output into C-corporations and pass-through businesses. It also supplies income and investment categories. For legal form $\ell \in \{C, P\}$, where C denotes C-corporations and P pass-throughs, define

$$a_\ell = \left\langle R\Big(B_t^{\ell,\mathrm{L}}, B_t^{\mathrm{L}}\Big) \right\rangle_{\mathcal{W}_B}, \qquad B_t^\ell = a_\ell B_t. \tag{21}$$

The ratio convention R is defined in equation (2). These shares apply to all reported years, including the historical portion beginning in 2018. The legal-form output series therefore describe the allocation implied by the benchmark average and the current business-output path. The C-corporate and pass-through shares are calculated separately. If the benchmark categories exhaust positive business GDP in every calibration year and use a common retained sample, their average shares sum to one.

4.3 Income and investment allocation

For an income or investment category J and its assigned output base $H(J)$, define

$$a_J = \left\langle J_t^{\mathrm{L}}/H_t(J)^{\mathrm{L}} \right\rangle_{\mathcal{W}_B}, \qquad J_t = a_J H_t(J). \tag{22}$$

Infinite and missing ratios are excluded from the mean, including a zero denominator. This convention differs from the zero-over-zero convention for legal-form GDP shares. The complete list of economic bases is given in Table 3.

These allocations are proportional aggregate relationships. They preserve the benchmark's average category-to-output ratio. A change in an assigned output base changes its allocated amount by the same percentage when the coefficient is fixed. Profits are scaled directly from total business GDP, even when the profit category names one legal form. Sector wages are allocated separately from total wages and salaries in equation (19). The collection is consequently a set of projection bases, rather than a reconciliation that forces every independently projected income category to sum to one total.

The inventory valuation adjustment removes inventory gains or losses associated with price changes from the supplied national-accounts profit concept; the capital consumption adjustment aligns depreciation with national-accounts economic depreciation [1]. The benchmark must supply these adjusted amounts. The projection model transports their ratios and does not reestimate the underlying inventory or depreciation adjustments.

Income or investment category	Assigned output base
Domestic C-corporation profits, with inventory valuation and capital consumption adjustments	Total business GDP B_t
Proprietors' income of sole proprietorships and partnerships, with the same adjustments	Total business GDP B_t
Domestic pass-through corporation profits, with the same adjustments	Total business GDP B_t
Pass-through residential investment	Pass-through GDP B_t^P
Pass-through nonresidential investment	Pass-through GDP B_t^P
Net interest, separately for each legal form	That legal form's GDP
Wages and salaries, separately for each legal form	That legal form's GDP
Supplements to wages and salaries, separately for each legal form	That legal form's GDP
Taxes on production and imports less subsidies, plus business transfers, separately for each legal form	That legal form's GDP

Table 3: Bases for fixed-share business allocations. Inventory valuation and capital consumption adjustments place the supplied income measures on their national-accounts basis.

5 Interest rates and monetary interest income

5.1 Observed rates and the policy-rate path

The term-structure model uses ten Treasury constant-maturity yields: 3 and 6 months, and 1, 2, 3, 5, 7, 10, 20, and 30 years. The U.S. Treasury produces the underlying constant-maturity estimates; the Federal Reserve Board publishes them in H.15 [12]. Monthly observations, distributed as the corresponding GS series, are averaged within calendar year. A supplied missing value makes that year's maturity average unavailable. A partial year's average uses the months supplied.

The historical effective federal funds rate is averaged in the same way. Near-term projections use the Federal Open Market Committee (FOMC) median federal funds path in its Summary of Economic Projections (SEP). Where a year appears in both history and the SEP path, the historical annual value takes precedence. Multiple SEP observations assigned to the same projection year are averaged. The SEP targets refer to the end of a calendar year; this model uses them directly as annual rate regressors [13].

Let S be one year after the later of the final nonmissing observed federal funds year and the final nonmissing SEP projection year. If there is no near-term SEP observation, S follows the last observed federal funds year. Let F^{LR} and g^{LR} be the latest available FOMC longer-run medians for the federal funds rate and real GDP growth, respectively. They are supplied inputs, in percent, rather than estimated coefficients.

The designated baseline's total real GDP, denoted Q_t^a, supplies a forward trend:

$$\bar{g}_t^a = \frac{1}{n_t} \sum_{k=0}^{n_t-1} 100 \left(\frac{Q_{t+k}^a}{Q_{t+k-1}^a} - 1 \right), \qquad n_t = \min(10, L - t + 1). \tag{23}$$

For the long-run portion of the rate path,

$$F_t = F^{LR} + \theta(\bar{g}_t^a - g^{LR}), \qquad S \le t \le L. \tag{24}$$

The selected sensitivity is $\theta = 0.5$ percentage points of federal funds per percentage point of growth.

A one-point reduction in trend growth relative to the FOMC assumption therefore lowers this rate input by half a point. Rates through $S-1$ retain the historical or SEP values. The combined rate input must supply one finite value for every forecast year from the first reported forecast year t_0 through $\min(S-1, L)$; an interior gap supplies no defined rate regressor. The rule for t_0 is given below. If $S > L$, no year receives the long-run adjustment. Positive, finite real GDP levels from $S-1$ through L are required when $S \leq L$. At $t = L$, the trend consists of the single final growth rate.

This rule replaces the FOMC longer-run growth assumption with the model's annual-average-output growth trend. The SEP's original GDP concept is fourth-quarter-over-fourth-quarter growth [13]; the use of the annual growth trend is the model's long-run approximation. The relation directly determines the annual long-run rate target. All scenarios receive the same path derived from Q^a.

5.2 Cross-sectional yield curves

For maturity τ measured in years, the Nelson–Siegel representation is

$$A(\tau, \lambda) = \frac{1 - e^{-\lambda\tau}}{\lambda\tau}, \qquad C(\tau, \lambda) = A(\tau, \lambda) - e^{-\lambda\tau}, \tag{25}$$

$$y_t(\tau) = b_{1t} + b_{2t}A(\tau, \lambda_t) + b_{3t}C(\tau, \lambda_t) + e_t(\tau). \tag{26}$$

The factors b_{1t}, b_{2t}, b_{3t} represent level, slope, and curvature; λ_t governs decay; and $e_t(\tau)$ is a cross-sectional fitting residual. This functional form follows Nelson and Siegel [14]; the growth anchor and time-series projection rules are separate model choices.

For each annual cross-section, consider curvature-peak targets u in the grid $0.25, 0.75, \ldots, 29.75$ years. For each target, choose $\lambda(u)$ in $[0.001, 1]$ that maximizes $C(u, \lambda)$. Conditional on that decay, estimate the three factors by unweighted least squares over finite maturity observations. Among admissible candidates with $0 < b_{1t} < 20$, choose the lowest sum of squared residuals; the first candidate wins a tie. The economic fitting domain consists of annual cross-sections with at least three identifying finite maturities and an admissible curve. This procedure specifies the shape search as well as the cross-sectional loss.

5.3 Factor dynamics and forecast timing

The time-series sample uses the available joint annual factor and federal funds history after removing the earliest Treasury year. Years with missing level factor or missing federal funds rate are omitted. Successive retained annual observations form successive estimation periods. Its starting year is determined by that joint historical coverage.

Each factor follows a separate nonseasonal autoregressive integrated moving-average (ARIMA) model. For $j = 1, 2, 3$, write

$$b_{jt} = \beta_j F_t + m_j(t) + u_{jt}, \qquad \beta_3 = 0, \tag{27}$$

$$\phi_j(\mathcal{B})(1 - \mathcal{B})^{d_j} u_{jt} = \vartheta_j(\mathcal{B})\epsilon_{jt}. \tag{28}$$

Here $\mathcal{B}$ is the one-period lag operator, ϕ_j and ϑ_j are autoregressive and moving-average lag polynomials, d_j is the difference order, and ϵ_{jt} is a zero-mean innovation. The level and slope respond to contemporaneous federal funds; curvature has no rate regressor. A selected deterministic term $m_j(t)$ is a constant for $d_j = 0$, a linear drift for $d_j = 1$, or zero for $d_j = 2$.

The statistical selection rule uses successive 5-percent KPSS level-stationarity tests to choose $d_j \leq 2$. KPSS denotes the Kwiatkowski–Phillips–Schmidt–Shin test. For level and slope the tests use residuals from a preliminary regression on federal funds and a constant; for curvature they use the factor itself. Autoregressive and moving-average orders are each bounded by 5 and by one-third of the retained sample size. A stepwise search compares corrected Akaike information criteria (AICc), with mean or drift terms eligible only for the difference orders just stated. Gaussian maximum likelihood refines a conditional-sum-of-squares initialization; no seasonal or power transformation is applied. Appendix B specifies the test bandwidth, criterion, candidate moves, deterministic-term choices, and stopping rule. These choices implement an automatic-forecasting approach of the kind described by Hyndman and Khandakar [15].

Let h_F be the latest observed federal funds year and m_F the number of its monthly observations. The first reported forecast year is

$$t_0 = \begin{cases} h_F + 1, & m_F \geq 6, \\ h_F, & m_F < 6. \end{cases} \tag{29}$$

All eligible annual factor observations, including a latest partial year, enter the fitting sample. The subsequent sequence of conditional forecasts is assigned labels $t_0, \ldots, L$, using the federal funds regressors with those labels. Observed annual Treasury yields are retained for labels before t_0.

Forecasts use the conditional means of equation (28), with future innovations set to their zero means. Let $\bar{\lambda}$ be the mean fitted decay over the retained annual sample. For every forecast year,

$$\widehat{y}_t(\tau) = \widehat{b}_{1t} + \widehat{b}_{2t} A(\tau, \bar{\lambda}) + \widehat{b}_{3t} C(\tau, \bar{\lambda}). \tag{30}$$

The projected decay is constant even when factor forecasts change. The model produces a point path rather than drawing random yield outcomes. Every scenario uses the same estimated factors, dynamics, and projected yields in a run. The resulting rate response to trend output is mediated by the estimated federal-funds loadings of the level and slope factors.

5.4 Monetary interest received by persons

BEA NIPA table 7.11 supplies monetary interest received by persons, excluding imputed interest recorded separately in that table. The NIPA persons sector includes households, nonprofit institutions serving households, and owners of nonfinancial noncorporate businesses [5]. Let M_t^{H} be the historical nominal amount in billions. The model estimates its share of nominal GDP using the 3-month and 10-year annual Treasury yields, $r_t^3 = y_t(0.25)$ and $r_t^{10} = y_t(10)$:

$$\log(M_t^{\mathrm{H}}/N_t) = \alpha + \beta_1^M r_t^3 + \beta_2^M r_{t-1}^3 + \beta_3^M \frac{r_t^{10} + r_{t-1}^{10} + r_{t-2}^{10}}{3} + v_t. \tag{31}$$

Here α is the intercept, β_j^M are rate coefficients, and v_t is the log-share residual. Ordinary least squares uses every year from 1990 through the final BEA year h_M. Historical nominal GDP supplies the denominator before 2018. The estimation window requires complete yield coverage, including two preceding years for lags, and positive GDP and monetary interest. Lags follow calendar years.

History is retained through h_M. For later years, the output is

$$M_t = N_t \exp\left[\widehat{\alpha} + \widehat{\beta}_1^M r_t^3 + \widehat{\beta}_2^M r_{t-1}^3 + \widehat{\beta}_3^M \frac{r_t^{10} + r_{t-1}^{10} + r_{t-2}^{10}}{3}\right]. \tag{32}$$

The projection lies directly on the exponentiated fitted log-share equation. Neither the final historical residual nor a lognormal retransformation adjustment is added. Thus this is the equation-implied share, rather than an explicitly bias-adjusted expectation of the level. The fitted share can differ from the final observed share at the history–projection boundary.

The equation is refitted for each scenario, using its assembled GDP history and the common yield path. When historical inputs coincide, the estimated coefficients coincide. Holding yields and coefficients fixed, a one-percent change in nominal GDP changes projected monetary interest by one percent. Yield changes affect the share through current short rates, lagged short rates, and the trailing three-year long rate. This lag structure carries an interest-rate change into subsequent years.

5.5 Reported quantities and horizons

The output is an annual collection of national aggregates. Table 4 describes the reporting spans; the final projection year is L. A missing value before a series' stated starting year denotes unavailable coverage.

Quantity	First reported year or span
Total and business GDP, real GDP, fiscal-year GDP; legal-form output and business accounts; wages, employment, population; monetary interest	2018
GDP deflator, PCE price index, CPI-U, CPI-W	1947
R-CPI-E and annual C-CPI-U	2018
Unadjusted CPI-U fiscal-year index	1987
Chained fiscal-year index and both chained tax-window measures	2001
Unadjusted CPI-U tax-window measure	1947
Multifactor productivity	First supplied Microsim year
Treasury yields	Available annual history within the combined reporting span, then t_0 through L

Table 4: Reporting horizons. Historical series supply the calibration data even where their reported output begins later.

A Implementation components

A.1 Assembly, inputs and aggregate outputs

Model boundary and selected settings. The assembly combines one or more supplied Microsim scenarios with shared national-account, price, labor, and financial histories. A business benchmark supplies allocation coefficients. The selected horizon and index base year apply across the resulting annual output collection. One designated scenario supplies the growth trend used for common Treasury rates.

Economic observations and units. The input component aligns producer series by economic year and puts output, income, employment, and population into their stated units. Annual observations support level anchoring and calibration. Monthly consumer-price histories additionally support tax-indexing windows and their publication vintages.

A.2 Output, prices and labor

Output translation. The output component maps private activity into nominal total and business GDP. A separately scaled private path determines the nominal sector share used in real total output. The nominal–real ratio supplies the GDP deflator.

Consumer prices and annual timing. The price component projects consumption and consumer price indexes through historical inflation differentials. Separate constructions provide fiscal-year indexes and September–August tax-indexing windows. These measures feed spending and tax calculations with distinct timing requirements.

National labor aggregates. Employment, population, and wages retain observed national levels and then follow the supplied aggregate growth paths. Productivity retains the supplied index path under the selected normalization. Each quantity's terminal rule extends it through the common horizon.

A.3 Business accounts and interest rates

Business allocation. The business-account component allocates output to legal forms and scales income and investment categories by their benchmark output ratios. It supplies nominal bases suitable for subsequent tax and budget calculations.

Term structure and interest receipts. The financial component estimates yield-curve factors from observed maturities and projects them conditional on the common federal funds path. The growth anchor links the long-run rate input to designated-scenario real output. A separate fitted equation translates short and long Treasury rates into monetary interest received by persons as a share of nominal GDP.

B Selection of the factor dynamics

The automatic selection rule determines the law of motion used in equation (28). Its difference order and deterministic term matter for the long-run forecast. This appendix specifies those choices without fixing the estimated coefficients to a particular historical vintage.

Difference order. Let n be the number of retained, nonmissing annual factor observations. For the level and slope, remove the fitted constant and federal funds component by preliminary ordinary least squares. For curvature, use the factor itself. Starting at $d = 0$, apply the KPSS test of stationarity about a constant to the resulting sequence, then to successive differences after a rejection, up to $d = 2$. At a stage with m observations, the Bartlett long-run variance estimate uses $\lfloor 3\sqrt{m}/13 \rfloor$ lags. Reject only when the statistic exceeds the 5-percent critical value 0.463. A constant sequence stops differencing immediately. If the selected difference order makes the differenced rate regressor constant, reduce that order by one.

Candidate models and criterion. For the selected d, the autoregressive and moving-average orders satisfy

$$0 \le p, q \le \min(5, \lfloor n/3 \rfloor).$$

Let k_x equal one for a factor with the federal funds regressor and zero for curvature, and let c equal one when the eligible deterministic term is included. That term is a mean when $d = 0$, drift when $d = 1$, and unavailable when $d = 2$. The federal funds coefficient remains in every candidate for the first two factors.

With maximized Gaussian log likelihood $\widehat{\ell}$ and parameter count $K = p + q + k_x + c + 1$, including the innovation variance, compare

$$\mathrm{AICc} = -2\widehat{\ell} + 2K + \frac{2K(K+1)}{n - d - K - 1}. \tag{33}$$

Here $n - d$ is the effective sample for the correction. A zero denominator gives an infinite criterion; a negative denominator retains the algebraic expression. For $n \le 3$, use the uncorrected Akaike information criterion, $-2\widehat{\ell} + 2K$.

Ordered stepwise search. Begin with the eligible deterministic term enabled. Evaluate seeds in the order $(2,2)$, $(0,0)$, $(1,0)$, $(0,1)$, bounding the first pair by the order limits and omitting infeasible one-lag seeds. For $n < 10$, use $(1,1)$ as the initial pair before applying those bounds. When a mean or drift is eligible, also evaluate $(0,0)$ without it. Strictly lower criterion values replace the best fit; ties preserve the earlier fit.

The best seed's order pair starts the local search. The search's deterministic-term setting initially remains enabled, including when the seed without that term has the best criterion. At the current (p, q), inspect feasible, previously untried changes in this order:

$$(-1,0),\ (0,-1),\ (1,0),\ (0,1),\ (-1,-1),\ (-1,1),\ (1,-1),\ (1,1).$$

Next consider toggling the eligible deterministic term at the current orders. Adopt the first strict improvement and restart the sequence; an improving toggle changes the deterministic-term setting. Stop at a neighborhood without an improvement or after 94 candidate fits, counting the seeds. Retain the best fit encountered. This is a local search with separate bounds on p and q, rather than a restriction on their sum.

Estimation and admissibility. Gaussian likelihood estimation uses conditional-sum-of-squares starting values and stationary/invertible autoregressive and moving-average parameterizations. Failed fits and fits with undefined coefficient standard errors receive an infinite criterion. A further screen rejects autoregressive or moving-average roots of modulus below 1.01 when the corresponding polynomial has effective degree at least two; effective degree is its last coefficient exceeding 10^{-8} in absolute value. This additional screen does not apply to a polynomial of effective degree one.

A constant factor is represented by a constant mean without an order search. An exactly constant differenced residual sequence also bypasses the search, using $(0, d, 0)$. With federal funds present, this model includes the rate regressor and an intercept only when $d = 0$. Without that regressor, a constant first difference supplies drift; a constant second difference uses no deterministic term. These rules specify the degenerate cases separately from the ordinary stepwise fit.

References

[1] Bureau of Economic Analysis. *NIPA Handbook*, Chapter 2: Fundamental Concepts, December 2024. https://www.bea.gov/resources/methodologies/nipa-handbook/pdf/chapter-02.pdf.

[2] Bureau of Economic Analysis. *NIPA Handbook*, Chapter 4: Estimating Methods, December 2023. https://www.bea.gov/resources/methodologies/nipa-handbook/pdf/chapter-04.pdf.

[3] Bureau of Economic Analysis. *NIPA Handbook*, Chapter 5: Personal Consumption Expenditures, December 2024. https://www.bea.gov/resources/methodologies/nipa-handbook/pdf/chapter-05.pdf.

[4] Bureau of Economic Analysis. *National Income and Product Accounts*, Table 2.1, population footnote. https://apps.bea.gov/iTable/?reqid=19&step=3&isuri=1. Series B230RC distributed as POPTHM: https://fred.stlouisfed.org/series/POPTHM.

[5] Bureau of Economic Analysis. *NIPA Handbook*, Chapter 14: Interest Paid and Received. https://www.bea.gov/resources/methodologies/nipa-handbook/pdf/chapter-14.pdf.

[6] Office of Management and Budget. *Historical Tables*, FY 2027 Budget, Table 10.1. https://www.whitehouse.gov/wp-content/uploads/2026/04/hist10z1_fy2027.xlsx. Fiscal-year GDP series distributed as FYGDP: https://fred.stlouisfed.org/series/FYGDP.

[7] Bureau of Labor Statistics. *Consumer Price Index Frequently Asked Questions*. https://www.bls.gov/cpi/questions-and-answers.htm.

[8] Bureau of Labor Statistics. *Research Consumer Price Index for Older Americans*. https://www.bls.gov/cpi/research-series/r-cpi-e-home.htm.

[9] Bureau of Labor Statistics. *Current Population Survey: Concepts and Definitions*. https://www.bls.gov/cps/definitions.htm.

[10] Bureau of Labor Statistics. *Handling Missing 2025 Consumer Expenditure Data*. https://www.bls.gov/cpi/additional-resources/handling-missing-2025-ce-data.htm.

[11] U.S. House of Representatives, Office of the Law Revision Counsel. *26 U.S.C. §1(f): Adjustments in Tax Tables So That Inflation Will Not Result in Tax Increases*. https://uscode.house.gov/us/usc/119-134/t26/s1/.

[12] Board of Governors of the Federal Reserve System. *H.15 Selected Interest Rates*. https://www.federalreserve.gov/releases/h15/.

[13] Board of Governors of the Federal Reserve System. *Guide to the Summary of Economic Projections*. https://www.federalreserve.gov/monetarypolicy/guide-to-the-summary-of-economic-projections.htm.

[14] Nelson, Charles R., and Andrew F. Siegel. 1987. Parsimonious Modeling of Yield Curves. *Journal of Business* 60(4): 473–489. https://doi.org/10.1086/296409. Working-paper version: https://www.nber.org/papers/w1594.

[15] Hyndman, Rob J., and Yeasmin Khandakar. 2008. Automatic Time Series Forecasting: The forecast Package for R. *Journal of Statistical Software* 27(3): 1–22. https://www.jstatsoft.org/article/view/v027i03.